

**Activity 1: Axioms of \mathbb{R}**

State precisely all the axioms of \mathbb{R} .

Activity 2: Uniqueness Proofs

- (a) **Uniqueness of additive inverse.** Prove: If $w \in \mathbb{R}$ satisfies $x + w = 0$ and $w + x = 0$ for all $x \in \mathbb{R}$, then $w = -x$.
- (b) **Uniqueness of multiplicative inverse.** Prove: If $y \in \mathbb{R}$ is any real number with the property that $xy = 1$ and $yx = 1$ for all $x \in \mathbb{R}$ with $x \neq 0$, then $y = 1/x$.

Activity 3: Contrapositive

Consider the statement: “If a^2 is even, then a is even.”

- (a) State the **contrapositive** of the above statement.
- (b) Prove the statement in part (a).

Activity 4: Contradiction

Consider the statement: “ $\sqrt[3]{2}$ is irrational.”

- (a) “Outline” **proof by contradiction**. That is, state the assumptions and explain what the goal is.
- (b) Provide a complete proof to the original statement.