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Activity 1: Linear Combination

Suppose that $\vec{u} = \langle 3, -2, 6, -4, 1 \rangle$, and $\vec{v} = \langle 4, -3, 8, -5, 0 \rangle$.
Is it possible to write $\langle 4, -1, 8, -7, 8 \rangle$ as a **linear combination** of $\vec{u}$ and $\vec{v}$? If so, how?

Activity 2: Parallel Vectors in $\mathbb{R}^2$

The goal of this item is to show that if $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ are vectors in $\mathbb{R}^2$, then, $\vec{u}$ and $\vec{v}$ are **parallel** to each other **if and only if** $u_1v_2 - u_2v_1 = 0$.

(a) Begin by stating the definition of $\vec{u}$ and $\vec{v}$ being parallel to each other (note: in our textbook/class slides....not the above statement!).

(b) ($\Longrightarrow$) Prove the forward implication: Show that if $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ are parallel to each other, then $u_1v_2 - u_2v_1 = 0$. Hint: just use direct substitution using (a).

(c) ($\Longleftarrow$) Prove the backward implication: Show that if $u_1v_2 - u_2v_1 = 0$, then $\vec{u} = a \cdot \vec{v}$ for some $a \in \mathbb{R}^2$ or $\vec{v} = b \cdot \vec{u}$ for some $b \in \mathbb{R}^2$. In other words, to reach this conclusion, you must be able to find the value of a or b.

Hint: do a Case-by-Case Analysis with Case 1: $u_1 \neq 0$, and Case 2: $u_1 = 0$. Recall that $\vec{0}_2$ is parallel to all vectors in $\mathbb{R}^2$, and a non-zero number has a **reciprocal** (Recall the axioms of $\mathbb{R}$). Case 2 will have sub-cases: Case 2a: $u_2 \neq 0$, and Case 2b: $u_2 = 0$.

Be sure to explain carefully in Case 2a why $\langle 0, u_2 \rangle$ is parallel to $\langle 0, v_2 \rangle$.

Activity 3: Contrapositive

Write down the **contrapositive** of the Theorem in the previous Activity.

Activity 4:

Use the previous item to show that if $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ are vectors in $\mathbb{R}^2$ that are **not** parallel to each other, then any vector $\langle x, y \rangle \in \mathbb{R}^2$ can be written as a linear combination of $\vec{u}$ and $\vec{v}$.

In other words, you have to show that for any $\langle x, y \rangle \in \mathbb{R}^2$, we can solve the vector equation $\langle x, y \rangle = r\vec{u} + s\vec{v}$ for r and s. You will have to do a Case-by-Case Analysis. We suggest the cases: (1) neither u_1 nor u_2 is zero, and (2) either u_1 or u_2 is zero (explain why they cannot both be zero).