MATH 10 - Linear Algebra — Fall 2019

Ch 1 — In-class Assignment #3

§1.4, 1.5

Dr. Jorge Basilio

Pasadena City College

gbasilio@pasadena.edu

9_16_2019

Activity 1: Definitions

Write the precise definitions of the following terms:

(a) a **linear combination** of the vectors $\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{R}^n$.

(b) the **span** of a set of vectors $S = \{\vec{v}_1, \ldots, \vec{v}_k\}$

Activity 2:

You are given the following set of 5 vectors from $\mathbb{R}^4$:

$$S = \{\vec{v}_1, \ldots, \vec{v}_5\} = S = \{\langle 2, -3, 4, -5\rangle, \quad \langle 1, -2, 2, -3\rangle, \quad \langle 1, 2, 2, 1\rangle, \quad \langle 5, -3, 7, -6\rangle, \quad \langle 6, 7, 3, 7\rangle\}$$

and $\vec{b} = \langle 11, 15, 1, 18\rangle \in \mathbb{R}^4$.

(a) Form the augmented matrix $[A|b]$.

(b) Use CoCalc/`SageMath` to compute the RREF of the augmented matrix.

(c) Just by looking at the RREF, explain why $\vec{b} \in \text{Span}(S)$.

(d) Write down all the solutions $x_1, \ldots, x_5$ to:

$$x_1\vec{v}_1 + x_2\vec{v}_2 + \cdots + x_5\vec{v}_5 = \vec{b}.$$

Identify all the leading and free variables.

(e) By setting the free variables to zero, find the "simplest" solution to the equation in (d), and write your answer by rewriting the equation above with the coefficients substituted in. The answer involves only two of the five vectors in S.

(f) Can we write $\vec{b}$ as a linear combination of only $\vec{v}_1, \vec{v}_2$, and $\vec{v}_5$? If this is possible, give an example.

(g) Can we write $\vec{b}$ as a linear combination of only $\vec{v}_3, \vec{v}_4$, and $\vec{v}_5$? If this is possible, give an example.

(h) Can we write $\vec{b}$ as a linear combination of only $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$? If this is possible, give an example.