MATH 10 - Linear Algebra — Fall 2019

Ch 1 Euclidean Spaces — In-class Assignment #4

§1.5, 1.6, 1.7, 1.8

Dr. Jorge Basilio

PASADENA CITY COLLEGE

gbasilio@pasadena.edu

10_2_2019

Activity 1: Definitions

Write the precise definitions of the following terms:

(a) a **linearly dependent** set of vectors $S = \{\vec{v}_1, \ldots, \vec{v}_k\}$.

(b) a **linearly independent** set of vectors $S = \{\vec{v}_1, \ldots, \vec{v}_k\}$.

(c) a **subspace** W of $\mathbb{R}^n$. Also, the **trivial subspaces** of $\mathbb{R}^n$.

(d) a **basis** of a non-zero subspace W of $\mathbb{R}^n$.

(e) Given a finite set X, the **cardinality** of X, $\text{card}(X)$.

(f) the **dimension** of a subspace W of $\mathbb{R}^n$.

(g) the **four fundamental subspaces** of a matrix $A_{m\times n}$. Be sure to indicate the ambient spaces.

(h) The **rank** and **nullity** of a matrix $A_{m\times n}$.

Activity 2: Proof

We would like to prove the following fact:
A non-empty subset $U \subseteq \mathbb{R}^n$ is a subspace of $\mathbb{R}^n$ **if and only if** $U = \text{Span}(S)$ for some non-empty subset S of $\mathbb{R}^n$.

(a) ($\Longrightarrow$) Give a statement of the “Forward direction”.

(b) ($\Longleftarrow$) Give a statement of the “Backward direction”.

(c) Which direction is the “easy” direction to prove? Explain why, or give a proof.

(d) Prove the other direction.

Activity 3: Computation

Consider the augmented matrix:

$$\left[A|\vec{b}\right] = \left[\begin{array}{cccccc|c} 3 & -5 & 1 & -3 & -3 & 4 & 3 \\ 4 & -6 & 4 & -2 & -3 & 2 & 8 \\ 1 & -5 & -13 & -11 & -6 & 18 & -19 \\ -2 & 4 & 2 & 4 & 2 & 2 & 0 \\ 26 & -38 & 30 & -10 & -23 & 48 & 48 \end{array}\right]; \quad \text{RREF: } R = \left[\begin{array}{cccccc|c} 1 & 0 & 7 & 4 & 0 & 5 & 8 \\ 0 & 1 & 4 & 3 & 0 & 7 & 3 \\ 0 & 0 & 0 & 0 & 1 & -8 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(a) Find a **basis** for the rowspace of A, $RS(A)$, using the information in R. (Reminder: A is only the first 6 columns)

(b) Find a **basis** for the columnspace of A, $CS(A)$.

(c) Find a **basis** for the nullspace of A, $NS(A)$. (You can use the "sight-seeing" mentioned in the textbook if you wish. Just be careful with free variables!)

(d) Express the non-basis columns of A as linear combinations of the basis vectors that you chose in part (b). (*Hint: what I call "magic"*)

(e) Describe all the solutions to $A\vec{x} = \vec{b}$ in the form $\vec{x}_P + \vec{x}_h$ where $\vec{x}_P$ is a particular solution, and $\vec{x}_h$ represents solutions to the homogenous system.

(f) Express row 3 of A as a linear combination of your basis in part (a).

Now, you are also given that RREF($A^\top$) is R' and:

$$R' = \begin{bmatrix} 1 & 0 & 7 & 0 & 8 \\ 0 & 1 & -5 & 0 & 3 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(Notice we don't include $\vec{b}$ in the transpose.)

(g) Find a **basis** for the nullspace of $A^\top$, $NS(A^\top)$.

(h) State the Dimension Theorem for Matrices, in general. (Don't forget it is both parts!)

(i) State the rank and the nullity of A and also $A^\top$, and verify the Dimension Theorem for A and $A^\top$.

(j) Now, use the information in R' to find *another* basis for $RS(A)$. (Hint: the rows of A become the columns of $A^\top$.)

(k) Express the 3rd row of A as a linear combination of the basis you found in part (j). (You'll need to set-up a SOE and use GJE to arrive at your answer)

(l) What is the nullity of $A^\top$? Explain how you got your answer. *(Hint: Warning! Do not find $RREF(A^\top)$. You don't need it!)*