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Activity 1: Definitions

Write the precise definitions of the following terms:

(a) when two vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$ are **orthongonal**.

(b) the **orthogonal complement** $W^{\perp}$ of a subspace W of $\mathbb{R}^n$.

(c) a **linear transformation** $T : \mathbb{R}^n \to \mathbb{R}^m$.

Activity 2: Proof

Suppose that $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation. We'd like to prove that $T(\vec{0}_n) = \vec{0}_m$.

(a) Prove it using the Equivalence of Linear Transformations and Matrices Theorem (recall this says: T is a LT iff $T = [A]$ for some $m \times n$ matrix)

(b) Prove it directly from the definition of a linear transformation using the Additivity Property. Hint: Consider $T(\vec{0}_n + \vec{0}_n)$ and use the properties of vector arithmetic from Ch 1 (this proof is similar in spirit to proofs from Axioms).

(c) Prove it directly from the definition of a linear transformation using the Homogeneity Property. Hint: How can the idea of part (b) be adapted? Hint: Use the properties of vector arithmetic from Ch 1 (this proof is similar in spirit to proofs from Axioms).

Activity 3: Computation

Let $A = \begin{bmatrix} 2 & -1 & -5 & 0 \\ 1 & 3 & 1 & 6 \\ 4 & 0 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 6 & 0 \\ 4 & 1 & 1 \\ -1 & -5 & 0 \\ 1 & -1 & 1 \end{bmatrix}$. Compute the following, if possible. If it is impossible, explain why not.

(a) AB

(b) BA

(c) $A + B^{\top}$.

Show all your STEPS! Indicate the size of your matrices in the lower, right-hand corner and underline or box the rows and columns to indicate the dot products of matrix multiplication.

Activity 4: Computation

Suppose that:

$$\begin{aligned} S &= \{\vec{w}_1, \vec{w}_2, \vec{w}_3, \vec{w}_4, \vec{w}_5, \vec{w}_6\} \\ &= \{\langle 3,5,-4,3,3,5\rangle, \langle -2,-4,6,-4,-2,0\rangle, \langle 1,-1,12,-7,-1,-1\rangle, \\ &\qquad \langle 5,6,5,-2,3,4\rangle, \langle 0,-1,5,-3,-1,-3\rangle, \langle 4,6,-2,2,4,10\rangle\} \subset \mathbb{R}^6 \end{aligned}$$

Let $W = Span(S)$. The vectors in S are assembled into the **rows** of a matrix A, and its RREF R is shown below. They are also assembled into the **columns** of a matrix, and its RREF R' is also shown below:

$$R = \begin{bmatrix} 1 & 0 & 7 & -4 & 0 & 2 \\ 0 & 1 & -5 & 3 & 0 & -5 \\ 0 & 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } R' = \begin{bmatrix} 1 & 0 & 0 & 1 & -1/2 & 2 \\ 0 & 1 & 0 & -1/2 & -1/2 & 1 \\ 0 & 0 & 1 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a **basis** for W using the information in R.

(b) Find a **basis** for W using the information in R'.

(c) Find a **basis** for $W^{\perp}$. Which RREF should you use?

The bases in (a) through (c) should have integer components.

Now, use the Two-for-One Theorem to decide if the following sets of vectors also form a basis for W. In other words, check that the sets have the correct number of vectors, and these vectors are linearly independent. You may use only the information in R and R'. The computations should be very simple.

(d) $\{\vec{w}_2, \vec{w}_3, \vec{w}_4\}$

(e) $\{\vec{w}_1, \vec{w}_2, \vec{w}_5, \vec{w}_6\}$

(f) $\{\vec{w}_2, \vec{w}_4, \vec{w}_5\}$

(g) $\{\vec{w}_2, \vec{w}_6\}$

(h) $\{\vec{w}_1, \vec{w}_2, \vec{w}_6\}$

Activity 5: Computation

Let $T_1 : \mathbb{R}^3 \to \mathbb{R}^4$ and $T_2 : \mathbb{R}^4 \to \mathbb{R}^2$,

$$T_1(\langle x,y,z\rangle) = \langle 2x-3y, 5y-7z, x-y+4z, 6x+y-z\rangle$$

$$T_2(\langle x_1,x_2,x_3,x_4\rangle) = \langle 5x_1+2x_3-x_4, 2x_1+8x_2-6x_3+7x_4\rangle$$

(a) Find $[T_1]$ and its dimension.

(b) Find $[T_2]$ and its dimension.

(c) Compute $T_2 \circ T_1$ using the definition.

(d) Compute $[T_2 \circ T_1]$. Hint: use matrix multiplication.