# Ch 3 Vector Spaces — In-class Assignment #7

§3.1, 3.2

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## Activity 1: Definitions

Write the precise definitions of the following terms:

(a) list the 10 axioms of a **vector space** $(V, \oplus, \odot)$.

(b) the **span** of a set of vectors $S = \{\vec{v}_i \mid i \in I\}$ of a vector space $V$.

(c) the **linear independence** of a set of vectors $S = \{\vec{v}_i \mid i \in I\}$ of a vector space $V$.

(d) a **subspace** $W$ of $V$.

(e) a **basis** and the **dimension** of a non-zero subspace $W$ of a vector space $(V, \oplus, \odot)$.

## Activity 2: Computation

Let $V = \mathbb{R}^2$ where we change both vector addition and scalar multiplication to:

$$\langle x_1, y_1\rangle \oplus \langle x_2, y_2\rangle = \langle x_1 + x_2 - 1, y_1 + y_2 + 2\rangle$$

and

$$k \odot \langle x, y\rangle = \langle kx + 1, ky - 2\rangle.$$

(a) Warm-up: Compute $\langle 4, -3\rangle \oplus \langle 2, 5\rangle$ and $-3 \odot \langle 7, -2\rangle$.

(b) Is there a **zero vector** for $V$? *(Hint: Let $\vec{z} = \langle a, b\rangle$ and attempt to solve the equations $\vec{z} \oplus \vec{v} = \vec{v}$ and $\vec{v} \oplus \vec{z} = \vec{v}$, for all $\vec{v} \in \mathbb{R}^2$.)*

(c) Are there **additive inverses**, $-\vec{v}$?

(d) Does the **distributive rule** $(r + s) \odot \vec{v} =$ ???? hold????

(e) Does the **distributive rule** $r \odot (\vec{v} \oplus \vec{w}) =$ ???? hold????

## Activity 3: Computation

Consider the infinite set:

$$S = \left\{xe^{-x}, x^2e^{-x/2}, x^3e^{-x/3}, \ldots\right\}$$

Assume that the "obvious" pattern continues. This is a subset of $\mathcal{F}(\mathbb{R}, \mathbb{R})$ (Actually, it is a subset of $\mathcal{C}^\infty(\mathbb{R}, \mathbb{R})$–the infinitely differentiable real-valued functions).

(a) Write this set using **set-builder** notation. What is your **indexing set**?

(b) Decide wether or not $S$ is linearly independent. Defend your answer.

## Activity 4: Computation

Suppose that $W$ be a subset of $V = \mathbb{P}^3$ defined by:

$$W = \{p(x) \in \mathbb{P}^3 \mid p(-2) = -p(3) \text{ and } p'(4) = 2p''(1)\}.$$

(a) Show that $W$ is a **subspace** of $\mathbb{P}^3$.
*(Hint: what three properties do you need to show?)*

(b) Find a basis for $W$.

(c) What is $\dim(W)$?

## Activity 5: Proof

Let $V = (V, \oplus, \odot)$ be a vector space. Note: you do not need to use the $\oplus, \odot$ notation below. You may if you wish but it's not required.

(a) **Prove:** The additive identity is unique. That is, for all $\vec{v} \in V$, if there exists a $\vec{w}_v \in V$ such that $\vec{v} + \vec{w}_v = \vec{0}_V$ and $\vec{w}_v + \vec{v} = \vec{0}_V$, then $-\vec{v} = \vec{w}_v$.

(b) Let $r \in \mathbb{R}$, $\vec{v} \in V$. **Prove:** (Zero Factors Theorem) $r \odot \vec{v} = \vec{0}_V$ **if and only if** $r = 0$ OR $\vec{v} = \vec{0}_V$.