Eigentheory — In-class Assignment #9

§6.1–6.3

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Activity 1: Definitions

Write the precise definitions of the following terms: Let $A \in \mathbb{M}_{n\times n}$.

(a) **eigenvalue**, **eigenvector**, and **eigenspace** for a matrix A.

(b) **characteristic polynomial** and the **characteristic equation** for A.

(c) A is **diagonalizable**.

(d) **algebraic multiplicity** and **geometric multiplicity**.

Activity 2: Computation

Let

$$A = \begin{bmatrix} 7 & 0 & 0 & 0 \\ 15 & -3 & 0 & 0 \\ 6 & -4 & 7 & 0 \\ 21 & -14 & 35 & -3 \end{bmatrix}$$

(a) Find the **characteristic polynomial** $p_A(\lambda)$.

(b) Find the **eigenvalues**.

(c) Find a basis for each **eigenspace**.

(d) What is the **dimension** of each eigenspace?

(e) Determine whether or not the algebraic multiplicity equals the geometric multiplicity for each eigenspace.

(f) Find C and D so that $A = CDC^{-1}$, if possible. Arrange D so that the eigenvalues are increasing.

(g) Compute: A^4 without using technology (sci calc is ok, just no `SAGEMath`). Show work!

Activity 3: Computation

(a) Show that $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable.

(b) Which of the following matrices are diagonalizable, and why?

A. $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ C. $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ D. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Activity 4: Proofs

Let $A \in \mathbb{M}_{n\times n}$. Let $T_A : \mathbb{R}^n \to \mathbb{R}^n$ be the linear transformation associated to A, i.e. $T_A(\vec{v}) = A\vec{v}$ and $[T_A] = A$.

(a) **Prove:** A and $A^\top$ have exactly the same eigenvalues.

Hint: you may wish to prove first that they have the same characteristic polynomials.

(b) **Prove:** Suppose that A is invertible. Then $\lambda \in \mathbb{R}$ is an eigenvalue of A if and only if λ^{-1} is an eigenvalue of the inverse A^{-1}.

Hint: As part of your proof, explain why the expression λ^{-1} makes sense (hint: use that A is invertible).

(c) **Prove:** Eigenspaces are **invariant** under T. That is, show that if λ is an eigenvalue of A, then

$$T_A(\mathrm{Eig}(A, \lambda)) \subseteq \mathrm{Eig}(A, \lambda).$$

Hint: this is really short.