MATH 10 - Linear Algebra — Fall 2019

Axioms of Real Numbers — Handout

Sets, Axioms, Logic, & Proofs

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Axioms for Real Numbers

There exists a non-empty set $\mathbb{R}$ that satisfies the following axioms:

(A1) Closure Property of Addition: $\forall x, y \in \mathbb{R}$, $x + y \in \mathbb{R}$.

(A2) Closure Property of Multiplication: $\forall x, y \in \mathbb{R}$, $xy \in \mathbb{R}$.

(A3) Commutative Property of Addition: $\forall x, y \in \mathbb{R}$, $x + y = y + x$.

(A4) Commutative Property of Multiplication: $\forall x, y \in \mathbb{R}$, $xy = yx$.

(A5) Associative Property of Addition: $\forall x, y, z \in \mathbb{R}$, $x + (y + z) = (x + y) + z$.

(A6) Associative Property of Multiplication: $\forall x, y, z \in \mathbb{R}$, $x(yz) = (xy)z$.

(A7) Distributive Property of Multiplication over Addition: $\forall x, y, z \in \mathbb{R}$, $x(y + z) = xy + xz$.

(A8) Existence of the Additive Identity: $\exists\, 0 \in \mathbb{R}$ such that $\forall x \in \mathbb{R}$, we have $x + 0 = x$ and $0 + x = x$.

(A9) Existence of the Multiplicative Identity: $\exists\, 1 \in \mathbb{R}$ such that $\forall x \in \mathbb{R}$, we have $x1 = x$ and $1x = x$.

(A10) Existence of the Additive Inverse: $\forall x \in \mathbb{R}$, $\exists(-x) \in \mathbb{R}$ such that $x + (-x) = 0$ and $(-x) + x = 0$.

(A11) Existence of the Multiplicative Inverse: $\forall x \in \mathbb{R}$, except $x = 0$, $\exists(1/x) \in \mathbb{R}$ such that $x(1/x) = 1$ and $(1/x)x = 1$.