

Good vs Bad Cheat Sheets

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GCSs vs BCSs

There are Good Cheat Sheets (GCSs) and there are Bad Cheat Sheets (BCSs).

Don't make BCSs! Make GCSs!

Inspiration of GCSs

VECTOR CALCULUS MATH 53M

CURL Curl $\vec{F} = \nabla \times \vec{F}$ $\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$	Curl $\vec{F} = \nabla \times \vec{F}$ For a conservative field $\text{Curl } \vec{F} = 0$	GRADIENT $\nabla \phi = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z} \right)$	DEL OPERATOR $\nabla^2 \phi = \nabla \cdot \nabla \phi$
DIVERGENCE $\text{div } \vec{F} = \nabla \cdot \vec{F}$	GRADIENT Direction of steepest increase	DIVERGENCE Net flow out of a volume	DEL OPERATOR Laplacian
IRROTATIONAL SURFACES $\nabla \cdot (\nabla \phi) = \nabla^2 \phi$	IRROTATIONAL $\text{Curl } (\nabla \phi) = 0$	IRROTATIONAL $\nabla \times (\nabla \phi) = 0$	IRROTATIONAL $\nabla \cdot (\nabla \phi) = \nabla^2 \phi$
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IRROTATIONAL $\nabla \times (\nabla \phi) = 0$	IRROTATIONAL $\nabla \cdot (\nabla \phi) = \nabla^2 \phi$	IRROTATIONAL $\nabla \times (\nabla \phi) = 0$	IRROTATIONAL $\nabla \cdot (\nabla \phi) = \nabla^2 \phi$

THE 5 THEOREMS OF CALCULUS

FUNDAMENTAL THEOREM OF CALCULUS $\int_a^b f(x) dx = F(b) - F(a)$	GREEN'S THEOREM $\oint_C \vec{F} \cdot d\vec{r} = \iint_R \text{Curl } \vec{F} \cdot \hat{n} dA$
STOKES'S THEOREM $\oint_C \vec{F} \cdot d\vec{r} = \iint_S \text{Curl } \vec{F} \cdot \hat{n} dA$	DIVERGENCE THEOREM $\oint_S \vec{F} \cdot \hat{n} dA = \iiint_V \text{div } \vec{F} dV$
HIERARCHY OF DIMENSION	HIERARCHY OF DIMENSION

ABSOLUTE VALUES

$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

FACTORIZING SPECIAL POLYNOMIALS

$A^2 - B^2 = (A+B)(A-B)$
 $A^2 + B^2 = (A+B)(A-B) + 2AB$
 $A^2 + B^2 = (A+B)(A-B) + 2AB$

LIMITS

Limits = 0
 $\lim_{x \rightarrow a} f(x) = 0$ when $\frac{1}{x-a} = 0$ and $f(x) > 0$ when $x > a$

Limits at 0
 $\lim_{x \rightarrow 0} f(x) = L$ if and only if $\lim_{x \rightarrow 0} f(x) = L = \lim_{x \rightarrow 0} f(x)$

LIMIT LAWS FOR USE IN PROOFS

- $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} [c f(x)] = c \lim_{x \rightarrow a} f(x)$
- $\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ if $\lim_{x \rightarrow a} g(x) \neq 0$

DERIVATIVE BY DEFINITION

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x) = \frac{df(x)}{dx}$

DERIVATION RULES

POWER RULE	PRODUCT RULE	QUOTIENT RULE	CHAIN RULE
$\frac{d}{dx} x^n = n x^{n-1}$	$\frac{d}{dx} (f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$	$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$	$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$

INTERMEDIATE VALUE THEOREM

Suppose f is continuous on the closed interval $[a, b]$ and let N be any number strictly between $f(a)$ and $f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

SQUEEZE THEOREM

If $f(x) \leq g(x) \leq h(x)$ for all x in an open interval that contains a (except possibly at a) and $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$. Then $\lim_{x \rightarrow a} g(x) = L$.

QUADRATIC FORMULA

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

PHYSICS

Distance
 $s = \int v dt$

Velocity
 $v = \frac{ds}{dt}$

Acceleration
 $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Projectile Motion
 $s = v_0 t + \frac{1}{2} a t^2$
 $v = v_0 + a t$

Work
 $W = \int \vec{F} \cdot d\vec{r}$

Energy
 $E = \int \vec{F} \cdot d\vec{r}$

Angular Motion
 $\theta = \int \omega dt$
 $\omega = \frac{d\theta}{dt}$
 $\alpha = \frac{d\omega}{dt}$

Rotational Kinematics
 $s = r\theta$
 $v = r\omega$
 $a = r\alpha$

Rotational Dynamics
 $\tau = r \times F$
 $L = I\omega$
 $\tau = \frac{dL}{dt}$

Simple Harmonic Motion
 $F = -kx$
 $m \frac{d^2x}{dt^2} = -kx$
 $x = A \cos(\omega t + \phi)$

Wave Motion
 $y = A \sin(kx - \omega t + \phi)$

Relativity
 $E = mc^2$
 $E = \gamma mc^2$
 $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Inspiration of GCSs

- <https://www.cfa.harvard.edu/~afriedman/CheatSheetsIndex.html>
- https://www.cfa.harvard.edu/~afriedman/Images/CheatSheetScans/MATH53_1.jpg

Horrors of BCs!

Derivatives: $\ln(u) = u^{-1} \cdot e^u = e^u \cdot u \cdot \cos u = \sin u \cdot u \cdot \tan u = \sec^2 u \cdot u \cdot \sec u = \sec u \tan u \cdot u \cdot \csc u = \csc u \cot u \cdot u \cdot \cot u = \csc^2 u \cdot u \cdot u$
Unit Circle: $0^\circ=0$ or 2π ; $(1,0)$ $30^\circ=\pi/6=(\sqrt{3}/2, 1/2)$ $45^\circ=\pi/4=(\sqrt{2}/2, \sqrt{2}/2)$ $60^\circ=\pi/3=(1/2, \sqrt{3}/2)$ $90^\circ=\pi/2=(0,1)$ $120^\circ=2\pi/3=(-1/2, \sqrt{3}/2)$ $135^\circ=3\pi/4=(-\sqrt{2}/2, \sqrt{2}/2)$ $150^\circ=5\pi/6=(-\sqrt{3}/2, 1/2)$ $180^\circ=\pi=(-1,0)$ $\text{Sine}=\text{Opp}/\text{Hyp}$ $\text{Cos}=\text{Adj}/\text{Hyp}$ $\text{Tan}=\frac{\text{Opp}}{\text{Adj}}$

Practice Problems: Differentiate $\ln(\cos x) \cdot (e^{\cos x})^2 \cdot \arctan(x-1) \cdot 4^x(x^2)$. $k(x)=\ln(\cos(x^2)) \cdot \ln(e^{(x^2+2)}) = 2\ln(\cos(x^2+2)) = 2\ln(\cos(x^2+2)) \cdot \cos(x^2+2) \cdot (-\sin(x^2+2)) \cdot 2x = -2x \sin(x^2+2) \cos(x^2+2) = -x \sin(2(x^2+2)) = -x \sin(2x^2+4) = -x \cdot 2 \cos(2x^2+4) \cdot 4x = -8x^2 \cos(2x^2+4)$

Compute the indefinite integrals $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C$ $\int \frac{1}{1+x^2} dx = \arctan(x) + C$ $\int \frac{1}{1-x^2} dx = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + C$ $\int \frac{1}{1+x^2} dx = \arctan(x) + C$ $\int \frac{1}{1-x^2} dx = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + C$ $\int \frac{1}{1+x^2} dx = \arctan(x) + C$ $\int \frac{1}{1-x^2} dx = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + C$

where $u = e^{(2x)} + 4$ and $du = 2e^{(2x)} dx$ or $1/2 du = e^{(2x)} dx$. So $\int \frac{1}{u} du = \ln|u| = \ln(e^{(2x)} + 4)$. **Compute the following definite integrals** $\int_0^1 \frac{1}{\sqrt{x}} dx = 2\sqrt{x} \Big|_0^1 = 2$ $\int_0^1 \frac{1}{x} dx = \ln|x| \Big|_0^1 = 0$ $\int_0^1 \frac{1}{x^2} dx = -\frac{1}{x} \Big|_0^1 = -1$ $\int_0^1 \frac{1}{x^3} dx = -\frac{1}{2x^2} \Big|_0^1 = -\frac{1}{2}$ $\int_0^1 \frac{1}{x^4} dx = -\frac{1}{3x^3} \Big|_0^1 = -\frac{1}{3}$ $\int_0^1 \frac{1}{x^5} dx = -\frac{1}{4x^4} \Big|_0^1 = -\frac{1}{4}$ $\int_0^1 \frac{1}{x^7} dx = -\frac{1}{6x^6} \Big|_0^1 = -\frac{1}{6}$ $\int_0^1 \frac{1}{x^9} dx = -\frac{1}{8x^8} \Big|_0^1 = -\frac{1}{8}$ $\int_0^1 \frac{1}{x^{11}} dx = -\frac{1}{10x^{10}} \Big|_0^1 = -\frac{1}{10}$ $\int_0^1 \frac{1}{x^{13}} dx = -\frac{1}{12x^{12}} \Big|_0^1 = -\frac{1}{12}$ $\int_0^1 \frac{1}{x^{15}} dx = -\frac{1}{14x^{14}} \Big|_0^1 = -\frac{1}{14}$ $\int_0^1 \frac{1}{x^{17}} dx = -\frac{1}{16x^{16}} \Big|_0^1 = -\frac{1}{16}$ $\int_0^1 \frac{1}{x^{19}} dx = -\frac{1}{18x^{18}} \Big|_0^1 = -\frac{1}{18}$ $\int_0^1 \frac{1}{x^{21}} dx = -\frac{1}{20x^{20}} \Big|_0^1 = -\frac{1}{20}$ $\int_0^1 \frac{1}{x^{23}} dx = -\frac{1}{22x^{22}} \Big|_0^1 = -\frac{1}{22}$ $\int_0^1 \frac{1}{x^{25}} dx = -\frac{1}{24x^{24}} \Big|_0^1 = -\frac{1}{24}$ $\int_0^1 \frac{1}{x^{27}} dx = -\frac{1}{26x^{26}} \Big|_0^1 = -\frac{1}{26}$ $\int_0^1 \frac{1}{x^{29}} dx = -\frac{1}{28x^{28}} \Big|_0^1 = -\frac{1}{28}$ $\int_0^1 \frac{1}{x^{31}} dx = -\frac{1}{30x^{30}} \Big|_0^1 = -\frac{1}{30}$ $\int_0^1 \frac{1}{x^{33}} dx = -\frac{1}{32x^{32}} \Big|_0^1 = -\frac{1}{32}$ $\int_0^1 \frac{1}{x^{35}} dx = -\frac{1}{34x^{34}} \Big|_0^1 = -\frac{1}{34}$ $\int_0^1 \frac{1}{x^{37}} dx = -\frac{1}{36x^{36}} \Big|_0^1 = -\frac{1}{36}$ $\int_0^1 \frac{1}{x^{39}} dx = -\frac{1}{38x^{38}} \Big|_0^1 = -\frac{1}{38}$ $\int_0^1 \frac{1}{x^{41}} dx = -\frac{1}{40x^{40}} \Big|_0^1 = -\frac{1}{40}$ $\int_0^1 \frac{1}{x^{43}} dx = -\frac{1}{42x^{42}} \Big|_0^1 = -\frac{1}{42}$ $\int_0^1 \frac{1}{x^{45}} dx = -\frac{1}{44x^{44}} \Big|_0^1 = -\frac{1}{44}$ $\int_0^1 \frac{1}{x^{47}} dx = -\frac{1}{46x^{46}} \Big|_0^1 = -\frac{1}{46}$ $\int_0^1 \frac{1}{x^{49}} dx = -\frac{1}{48x^{48}} \Big|_0^1 = -\frac{1}{48}$ $\int_0^1 \frac{1}{x^{51}} dx = -\frac{1}{50x^{50}} \Big|_0^1 = -\frac{1}{50}$ $\int_0^1 \frac{1}{x^{53}} dx = -\frac{1}{52x^{52}} \Big|_0^1 = -\frac{1}{52}$ $\int_0^1 \frac{1}{x^{55}} dx = -\frac{1}{54x^{54}} \Big|_0^1 = -\frac{1}{54}$ $\int_0^1 \frac{1}{x^{57}} dx = -\frac{1}{56x^{56}} \Big|_0^1 = -\frac{1}{56}$ $\int_0^1 \frac{1}{x^{59}} dx = -\frac{1}{58x^{58}} \Big|_0^1 = -\frac{1}{58}$ $\int_0^1 \frac{1}{x^{61}} dx = -\frac{1}{60x^{60}} \Big|_0^1 = -\frac{1}{60}$ $\int_0^1 \frac{1}{x^{63}} dx = -\frac{1}{62x^{62}} \Big|_0^1 = -\frac{1}{62}$ $\int_0^1 \frac{1}{x^{65}} dx = -\frac{1}{64x^{64}} \Big|_0^1 = -\frac{1}{64}$ $\int_0^1 \frac{1}{x^{67}} dx = -\frac{1}{66x^{66}} \Big|_0^1 = -\frac{1}{66}$ $\int_0^1 \frac{1}{x^{69}} dx = -\frac{1}{68x^{68}} \Big|_0^1 = -\frac{1}{68}$ $\int_0^1 \frac{1}{x^{71}} dx = -\frac{1}{70x^{70}} \Big|_0^1 = -\frac{1}{70}$ $\int_0^1 \frac{1}{x^{73}} dx = -\frac{1}{72x^{72}} \Big|_0^1 = -\frac{1}{72}$ $\int_0^1 \frac{1}{x^{75}} dx = -\frac{1}{74x^{74}} \Big|_0^1 = -\frac{1}{74}$ $\int_0^1 \frac{1}{x^{77}} dx = -\frac{1}{76x^{76}} \Big|_0^1 = -\frac{1}{76}$ $\int_0^1 \frac{1}{x^{79}} dx = -\frac{1}{78x^{78}} \Big|_0^1 = -\frac{1}{78}$ $\int_0^1 \frac{1}{x^{81}} dx = -\frac{1}{80x^{80}} \Big|_0^1 = -\frac{1}{80}$ $\int_0^1 \frac{1}{x^{83}} dx = -\frac{1}{82x^{82}} \Big|_0^1 = -\frac{1}{82}$ $\int_0^1 \frac{1}{x^{85}} dx = -\frac{1}{84x^{84}} \Big|_0^1 = -\frac{1}{84}$ $\int_0^1 \frac{1}{x^{87}} dx = -\frac{1}{86x^{86}} \Big|_0^1 = -\frac{1}{86}$ $\int_0^1 \frac{1}{x^{89}} dx = -\frac{1}{88x^{88}} \Big|_0^1 = -\frac{1}{88}$ $\int_0^1 \frac{1}{x^{91}} dx = -\frac{1}{90x^{90}} \Big|_0^1 = -\frac{1}{90}$ $\int_0^1 \frac{1}{x^{93}} dx = -\frac{1}{92x^{92}} \Big|_0^1 = -\frac{1}{92}$ $\int_0^1 \frac{1}{x^{95}} dx = -\frac{1}{94x^{94}} \Big|_0^1 = -\frac{1}{94}$ $\int_0^1 \frac{1}{x^{97}} dx = -\frac{1}{96x^{96}} \Big|_0^1 = -\frac{1}{96}$ $\int_0^1 \frac{1}{x^{99}} dx = -\frac{1}{98x^{98}} \Big|_0^1 = -\frac{1}{98}$ $\int_0^1 \frac{1}{x^{101}} dx = -\frac{1}{100x^{100}} \Big|_0^1 = -\frac{1}{100}$

Chapter 1: Organizational Design	Levels of Managers	Human Resource Management	Organizational Design	Decision Support Systems	Human Resource Management
<p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Structure: The arrangement of jobs and positions within an organization.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p>	<p>Top Management: Responsible for setting the organization's overall strategy and direction.</p> <p>Middle Management: Responsible for implementing the organization's strategy and direction.</p> <p>Operational Management: Responsible for managing the day-to-day operations of the organization.</p> <p>Supervisory Management: Responsible for managing the work of individual employees.</p>	<p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p>	<p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p>	<p>Decision Support Systems: Computer-based systems that help managers make decisions.</p> <p>Decision Support Systems: Computer-based systems that help managers make decisions.</p> <p>Decision Support Systems: Computer-based systems that help managers make decisions.</p> <p>Decision Support Systems: Computer-based systems that help managers make decisions.</p>	<p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p>
<p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p>	<p>Top Management: Responsible for setting the organization's overall strategy and direction.</p> <p>Middle Management: Responsible for implementing the organization's strategy and direction.</p> <p>Operational Management: Responsible for managing the day-to-day operations of the organization.</p> <p>Supervisory Management: Responsible for managing the work of individual employees.</p>	<p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p>	<p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p> <p>Organizational Design: The process of determining an organization's internal structure and reporting relationships.</p>	<p>Decision Support Systems: Computer-based systems that help managers make decisions.</p> <p>Decision Support Systems: Computer-based systems that help managers make decisions.</p> <p>Decision Support Systems: Computer-based systems that help managers make decisions.</p> <p>Decision Support Systems: Computer-based systems that help managers make decisions.</p>	<p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p> <p>Human Resource Management: The process of managing an organization's human resources.</p>

Horrors of BCSs!

- <http://komplexify.com/epsilon/2009/04/23/when-cheat-sheets-go-bad/>
- <http://i.imgur.com/k0pQ35F.jpg?1>
- <http://img.wonderhowto.com/img/original/16/80/63494460985769/0/634944609857691680.jpg> (reddit: http://www.reddit.com/r/EngineeringStudents/comments/1ax982/only_1_sheet_all_owed_for_exam_no_problem/)
- you get the idea...

GCSs in a nutshell

- neat, organized, legible
- hand-written (this is important!)
- material organized by subjects, not necessarily the order it was covered
- important formulas (don't forget prerequisite material!)
- important algebra tricks

BCSs in a nutshell

- messy, font too small, illegible
- no organization, just copied all the text or all the formals at random
- not hand-written (see second bullet)
- hard to find information

Making a GCS is a process

First, it helps to organize the material into sections (usually by chapters and sections, but not always). For example, group the limits material, group the continuity material together, group the derivative rules, group the derivative applications, etc etc. Next, you need to go through your notes and textbook highlighting key terms, definitions, special tricks, and sometimes the steps to follows. The key here is to use *short-cuts* to refresh your memory of a subject without having to be so formal about it (eg. writing the short-cuts $+C/-0=-\text{infinity}$, $C/+infinity=0$, etc). Sometimes you might include a solution to a tricky problem (eg. computing limits using the guidelines with a tricky algebra step). But don't go overboard here: only include a few--you should already know how to do the easy and medium level questions because you've practiced it lots and lots. You might want to include a section that recalls review material like algebra tricks (rationalize, exponent rules, solving various equations, etc).

After you've reviewed all the material, grouped it together intelligently, you are ready to write a nice, neat cheat sheet. *The key is organization (so you can find what you need, when you need it) and neatness.*

Benefits of GCS

- a natural part of the study process
- making one by hand aids memorization
- helps organize complex, difficult information
- helps put information *into context*
- helps reduce stress if allowed to use on exams
- and many more!

Good luck and make GCSs!

Created on September 30, 2014 for my Math 2610 class