

1.4 SOE "System of Equations"

$$\begin{cases} x = 6 \\ y = -2 \\ z = 1 \end{cases}$$

$$\begin{aligned} x + 0y + 0z &= 6 \\ 0x + y + 0z &= -2 \\ 0x + 0y + z &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} & x & y & z & b \\ 1 & 0 & 0 & & 6 \\ 0 & 1 & 0 & & -2 \\ 0 & 0 & 1 & & 1 \end{array} \right]$$

Identity Matrix I_3

$$\Leftrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ 1 \end{bmatrix}$$

$$\Leftrightarrow (A|b)$$

Goal of GJR (Gauss-Jordan Elimination Algorithm) Reduction Elimination Algorithm

Start $(A|b)$ $\xrightarrow[\text{EROs}]{\text{GJR}}$ $(I^*|c^*)$ "continue w/ back substitution"

alt "augmented matrix" or $(I_n|c^*)$ "done!"

Theorem The solution $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = C^* = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$ to the original SOE

How to do it?

- * use EROs
- * use "clever" algorithm.

EROs Elementary Row Operations

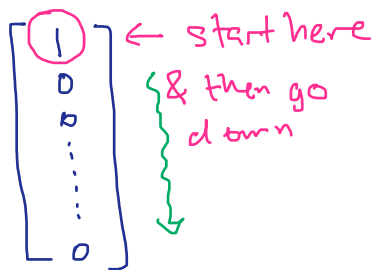
- ① swap any two rows $(R_i \leftrightarrow R_j)$
- ② multiply entire row by a non-zero scalar $(cR_i \rightarrow R_i) (c \neq 0)$
- ③ can replace any row by the sum of any two non-zero multiples of other rows.

$$(aR_i + bR_j \rightarrow R_i)$$

The Algorithm

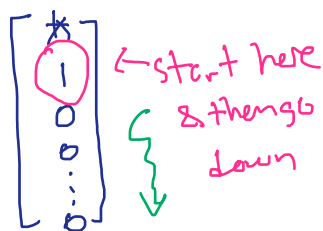
Start: $(A|b) = \left(\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & \dots & a_{mn} & b_m \end{array} \right)$

Step 1 turn column 1 into



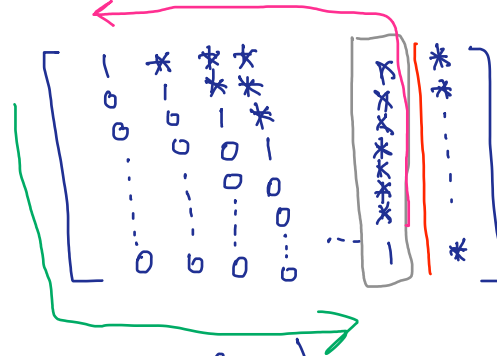
Using EROs.

Step 2 turn column 2 into



* = ignore

Step 3 continue in this way:

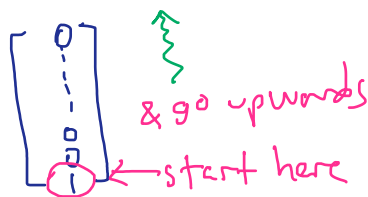


Using EROs.

If we stop here: call this REF (ROW echelon form)

Important Can stop here & solve using using "back substitution"

Step 4 "go upwards & backwards" make second to last column into



Steps continue in this pattern until:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & c_1 \\ 0 & 1 & 0 & 0 & c_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & c_n \end{array} \right]$$

fin.

This last step is called
RREF
(Reduced Row Echelon
Form)

Main Point

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

solves original SDEs!