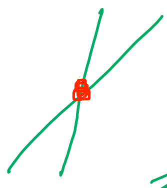
1.5

"square Matrix" $n \times n$: # rows = # Columns.

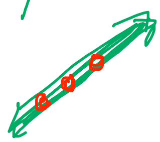
<u>Recall</u> 2×2 : consistent SOE: if Case I or II $n \times n$ cases same!

SOE { <u>Case I</u>



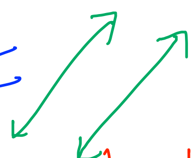
one solution $\bar{X} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ <u>one sol</u> $\bar{X} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$

<u>Case II</u>



∞ly many solutions ∞ly many sol

<u>Case III</u>



no solutions ($\emptyset$) no sol ($\emptyset$)

SOE <u>in</u>consistent

- Studying RREF to learn ~~the~~ case we're in.

<u>Case I</u> RREF = $\left[\begin{array}{ccccc|c} 1 & 0 & 0 & \cdots & 0 & * \\ 0 & 1 & 0 & & 0 & * \\ 0 & 0 & 1 & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & & \\ 0 & 0 & 0 & & 1 & * \end{array}\right]$ → one sol

identity matrix I_n

<u>Case II</u> RREF = must have at least one <u>entire row of 0s</u> $[0\,0 \cdots 0 \mid 0]$

↳ this corresponds to a "free variable"

→ ∞ly many sol

<u>Case III</u> RREF = must have at least one row of the form

$[0\,0 \cdots 0 \mid \text{non-zero\#}]$

<u>Thm</u> $S=\{\vec{v}\}$ is LI $\boxed{\text{iff}}$ $\vec{v}\neq\vec{0}$.

<u>Pf</u> $(\Rightarrow)$ Assume S is LI. WTS: $\vec{v}\neq\vec{0}$.

<u>Note</u> contrapositive:
if $\vec{v}=\vec{0}$ then $S=\{\vec{v}\}$ is LD.

This is true by previous theorem.

$(\Leftarrow)$ Assume $\vec{v}\neq\vec{0}$. WTS: S is LI.

Consider DTE: $\underline{x_1\vec{v}=\vec{0}}$.

<u>Use</u> Zero Factors Theorem: || Exercise 28, §1.1

$\underline{k\vec{v}=\vec{0}} \iff k=0$ or $\vec{v}=0$.

But we know $\vec{v}\neq\vec{0}$. We must have $k=0$. ie $x_1=0$.
ie $\vec{x}=\vec{0}$ only trivial sol solves DTE. So S is LI.

$\square$