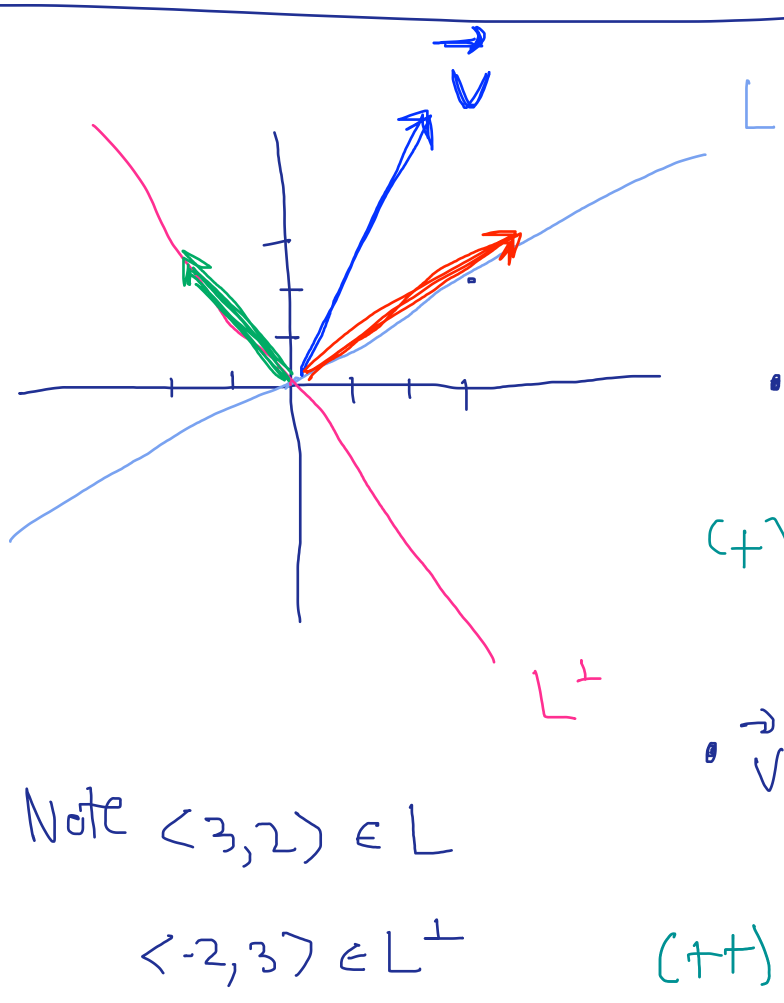
2.2

Example $L: y = \frac{2}{3}x$ $\quad L^\perp: y = \frac{-3}{2}x$

Find $proj_L(\vec{v})$, $proj_{L^\perp}(\vec{v})$,

$refl_L(\vec{v})$, $refl_{L^\perp}(\vec{v})$



$\star$ Key $\boxed{\vec{v} = proj_L(\vec{v}) + proj_{L^\perp}(\vec{v})}$

- $\vec{v} \in L$ then $proj_L(\vec{v}) \parallel \langle 3,2 \rangle$

(+) $\iff$ $\underline{proj_L(\vec{v}) = a\langle 3,2 \rangle}$

- $\vec{v} \in L^\perp$ then $proj_{L^\perp}(\vec{v}) \parallel \langle -2,3 \rangle$

Note $\langle 3,2 \rangle \in L$

$\langle -2,3 \rangle \in L^\perp$

(++) $\iff$ $\underline{proj_{L^\perp}(\vec{v}) = b\langle -2,3 \rangle}$

For some scalars $a, b \in \mathbb{R}$.

Use ($\star$) & (+) (++) :

$$\vec{v} \qquad \mathrm{proj}_L(\vec{v}) + \mathrm{proj}_{L^\perp}(\vec{v})$$

$$\langle x, y\rangle = a\langle 3,2\rangle + b\langle -2,3\rangle$$

$$\langle x, y\rangle = \langle 3a-2b, \; 2a+3b\rangle$$

given Find a, b.

SOE: $\begin{cases} 3a - 2b = x \\ 2a + 3b = y \end{cases}$

$$\left[\begin{array}{cc|c} 3 & -2 & x \\ 2 & 3 & y \end{array}\right] \xrightarrow[R_2 - 2R_1 \to R_2]{R_1 - R_2 \to R_1} \left[\begin{array}{cc|c} 1 & -5 & x-y \\ 0 & 13 & -2x+3y \end{array}\right]$$

$$\xrightarrow[\frac{1}{13}R_2 \to R_2]{} \left[\begin{array}{cc|c} 1 & -5 & x-y \\ 0 & 1 & \left(\frac{-2}{13}\right)x + \left(\frac{3}{13}\right)y \end{array}\right]$$

$$\xrightarrow[R_1 + 5R_2]{} \left[\begin{array}{cc|c} 1 & 0 & \left(\frac{3}{13}\right)x + \left(\frac{2}{13}\right)y \\ 0 & 1 & \left(\frac{-2}{13}\right)x + \left(\frac{3}{13}\right)y \end{array}\right]$$

$$\boxed{\begin{aligned} a &= \left(\frac{3}{13}\right)x + \left(\frac{2}{13}\right)y \\ b &= \left(\frac{-2}{13}\right)x + \left(\frac{3}{13}\right)y \end{aligned}}$$

So:

- $\text{proj}_L(\vec{v}) = a\langle 3,2\rangle = \left\langle \frac{9}{13}x + \frac{6}{13}y,\ \frac{6}{13}x + \frac{4}{13}y \right\rangle$

$$= \begin{bmatrix} \frac{9}{13} & \frac{6}{13} \\ \frac{6}{13} & \frac{4}{13} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \frac{1}{13} \begin{bmatrix} 9 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{[\text{proj}_L] = \frac{1}{13} \begin{bmatrix} 9 & 6 \\ 6 & 4 \end{bmatrix}}$$

- $\text{proj}_{L^\perp}(\vec{v}) = b\langle -2,3\rangle = \left\langle \frac{4}{13}x + \frac{-6}{13}y,\ \frac{-6}{13}x + \frac{9}{13}y \right\rangle$

$$= \frac{1}{13} \begin{bmatrix} 4 & -6 \\ -6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{[\text{proj}_{L^\perp}] = \frac{1}{13} \begin{bmatrix} 4 & -6 \\ -6 & 9 \end{bmatrix}}$$

- Using the vector eqs for reflections:

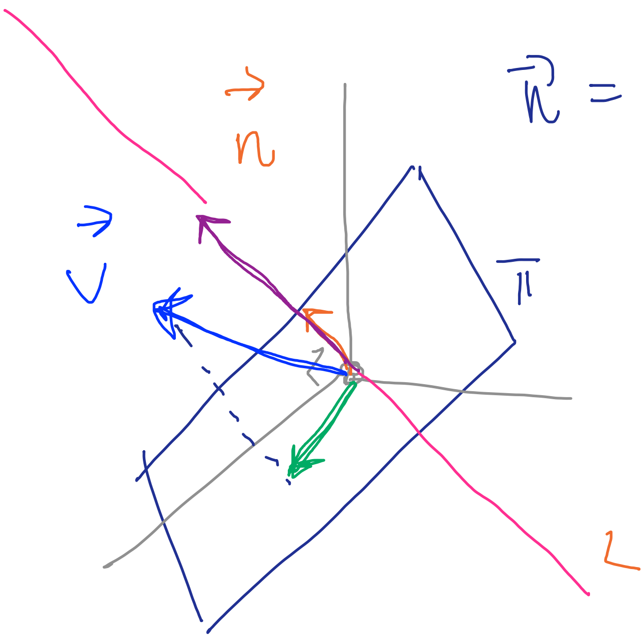
$$[\mathrm{Refl}_L] = \frac{1}{13}\begin{bmatrix} 5 & 12 \\ 12 & -5 \end{bmatrix}$$

$$[\mathrm{Refl}_{L^\perp}] = \frac{1}{13}\begin{bmatrix} -5 & -12 \\ -12 & 5 \end{bmatrix}$$

Ex Projection & Reflections in $\mathbb{R}^3$

$\Pi: \quad 3x - 5y + 2z = 0$

$\vec{n} = \langle 3, -5, 2 \rangle$



$\Pi^{\perp} = L = \mathrm{Span}(\vec{n})$

$= \{ t\vec{n} \mid t \in \mathbb{R} \}$

- Projection:

$$\boxed{\vec{v} = \mathrm{proj}_{\Pi}(\vec{v}) + \mathrm{proj}_L(\vec{v})}$$

Start w/ proj_L:

$\mathrm{proj}_L(\vec{v}) = t\vec{n}$ some $t \in \mathbb{R}$.

$= t\langle 3, -5, 2 \rangle$

$= \langle 3t, -5t, 2t \rangle$

Also: $\text{proj}_{\Pi}(\vec{v}) = \vec{v} - \text{proj}_L(\vec{v})$.

$$= \langle x, y, z \rangle - \langle 3t, -5t, 2t \rangle$$

$$= \langle x - 3t, y + 5t, z - 2t \rangle$$

Now, use $\Pi^{\perp} = L$ & $\text{proj}_{\Pi}(\vec{v}) \in \Pi$:

$$\underline{\vec{n} \circ \text{proj}_{\Pi}(\vec{v}) = 0}$$

$$\underline{3(x-3t) - 5(y+5t) + 2(z-2t) = 0}$$

x, y, z given, find t.

$$3x - 9t - 5y - 25t + 2z - 4t = 0$$

$$-38t \quad -3x + 5y - 2z$$

$$t = \left(\frac{3}{38}\right)x + \left(\frac{-5}{38}\right)z + \left(\frac{2}{38}\right)z$$

get: $\text{proj}_L(\vec{v}) = t\vec{n}$

$$= \left\langle \left(\frac{9}{38}\right)x + \left(-\frac{15}{38}\right)y + \left(\frac{6}{38}\right)z,\; \left(-\frac{15}{38}\right)x + \left(\frac{25}{38}\right)y + \left(-\frac{10}{38}\right)z,\; \left(\frac{6}{38}\right)x + \left(-\frac{10}{38}\right)y + \left(\frac{4}{38}\right)z \right\rangle$$

$$\boxed{\left[\operatorname{proj}_L\right] = \frac{1}{38}\begin{bmatrix} 9 & -15 & 6 \\ -15 & 25 & -10 \\ 6 & -10 & 4 \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Next find:

$$\operatorname{proj}_{\Pi}(\vec{v}) = \vec{v} - \operatorname{proj}_L(\vec{v})$$

$$= \langle x, y, z \rangle - \langle \;,\;,\; \rangle$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \frac{1}{38}\begin{bmatrix} 9 & -15 & 6 \\ -15 & 25 & -10 \\ 6 & -10 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \frac{1}{38}\begin{bmatrix} 29 & 15 & -6 \\ 15 & 13 & 10 \\ -6 & 10 & 34 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

$$\text{proj}_{\pi}(\vec{v}) = \left\langle \left(\frac{29}{38}\right)x + \left(\frac{15}{38}\right)y + \left(\frac{-6}{38}\right)z, \right.$$

$$\ldots\ldots,$$

$$\left. \ldots\ldots \right\rangle.$$