2.4

$T : \mathbb{R}^3 \to \mathbb{R}^4$

$\vec{v_1}$

$T(\langle 1,2,1\rangle) = \langle 3,5,2,-4\rangle$

$\vec{v_2}$

$T(\langle -1,1,0\rangle) = \langle 7,4,-3,8\rangle$

$\vec{v_3}$

$T(\langle 1,1,1\rangle) = \langle 2,1,5,6\rangle$

Use this to find:

$T(\langle 3,-5,8\rangle)$.

$\langle 3,-5,8\rangle \in \text{Span}(\{\vec{v_1},\vec{v_2},\vec{v_3}\})$ $\quad \vec{v_1},\vec{v_2},\vec{v_3} \in \mathbb{R}^3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 2 & 1 & 1 & -5 \\ 1 & 0 & 1 & 8 \end{array}\right] \xrightarrow[\text{RREF}]{\text{GJR}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -18 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 26 \end{array}\right]$$

Bonus: $v_1\ v_2\ v_3$ LI & Span make basis.

$\langle 3,-5,8\rangle = -18\vec{v_1} + 5\vec{v_2} + 26\vec{v_3}$

$T(\langle 3,-5,8\rangle) = -18T(\vec{v_1}) + 5T(\vec{v_2}) + 26T(\vec{v_3})$

$= -18\langle 3,5,2,-4\rangle$

$+ 5\langle 7,4,-3,8\rangle$

$+ 26\langle 2,1,5,6\rangle = \langle 33,-44,79,268\rangle$.