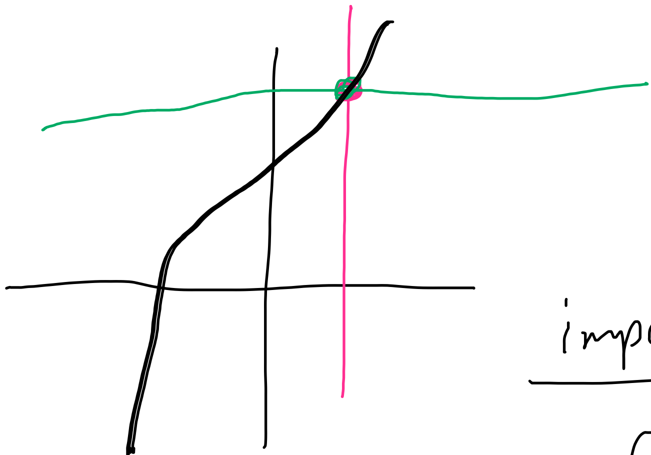
2.5

Motivation in $\mathbb{R}$



f passes VLT $\rightarrow$ f function

passes HLT $\rightarrow$ f is one-to-one

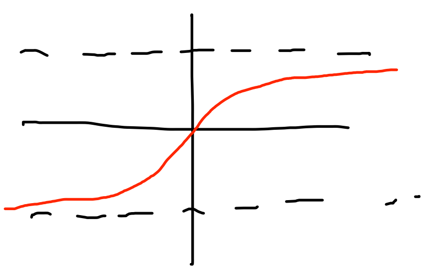
$1-1$

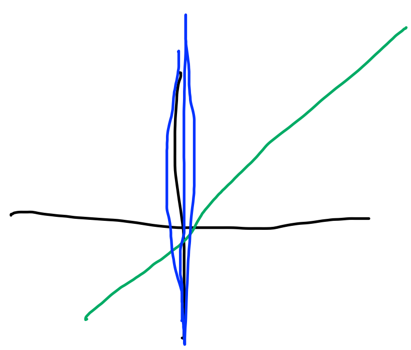
importance of 1-1

f has inverse!

$$x \xrightarrow{f} f(x) = y$$

$$x \xleftarrow{f^{-1}} y$$





Note: a line is 1-1 and onto

Main Q When does a LT $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ have an inverse?

Ans Need 2 properties: $1-1$ & onto

Ex Cont.

$\ker(T_1) = \text{Span}\left(\left\{\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\right\}\right).$

$\text{nullity}(T_1) = \dim(NS(T_1)) = \dim(\ker(T_1)) = 1.$

$\boxed{\text{Dimension Thm}}$ $\text{rank}(T_1) = n - \text{nullity}(T_1)$

$= 3 - 1 = 2.$

$\text{rank}(T_1) = \dim(\text{Range}(T_1))$

$= \dim(cs([T_1]))$

Reading R_1:

$CS([T_1]) = \text{Span}\left(\left\{\begin{bmatrix} 1 \\ 2 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \\ 4 \\ 7 \end{bmatrix}\right\}\right)$

$$\ker(T_2) = \{\vec{0}\}.$$

$$\text{nullity}(T_2) = \dim(\ker(T_2)) = 0.$$

$$\text{Dim Th:} \quad \text{rank}(T_2) = n - \text{nullity}(T_2)$$

$$= n$$

$$= 3$$

T_2: full rank

$$T_2 : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$$

$$\text{Range}(T_2) = CS([T_2])$$

$$= \text{Span}\left(\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -6 \\ -15 \\ 9 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \\ 4 \\ -7 \end{bmatrix} \right\} \right)$$