3.6 Coordinate Vectos and Matrices for Linear Transformations

Ex $T: \mathbb{P}^3 \to \mathbb{P}^2$ $\quad \boxed{T(p) = 3p' + 7x\,p'' + p(-1)\cdot x^2}$

$B = \{1, x, x^2, x^3\}$ $\qquad B' = \{1, x, x^2\}$

$$[T]_{B,B'} = \Big[\, [T(1)]_{B'} \;\Big|\; [T(x)]_{B'} \;\Big|\; [T(x^2)]_{B'} \;\Big|\; [T(x^3)]_{B'} \,\Big]$$

- $T(1) = 3\cdot 0 + 7x\cdot 0 + 1\cdot x^2$ $\qquad [T(1)]_{B'} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

  $= x^2 + 0\cdot x + 0\cdot 1$ (not in B')

  $= 0\cdot 1 + 0\cdot x + 1\cdot x^2$

  (in B')

- $T(x) = 3\cdot 1 + 7x(0) + (-1)\cdot x^2$ $\qquad [T(x)]_{B'} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

  $= 3\cdot 1 + 0\cdot x + (-1)\cdot x^2$

- $T(x^2) = 3(2x) + 7x(2) + (+1)\cdot x^2$ $\qquad [T(x^2)]_{B'} = \begin{bmatrix} 0 \\ 20 \\ 1 \end{bmatrix}$

  $= 0\cdot 1 + 20x + 1\cdot x^2$

- $T(x^3) = 3(3x^2) + 7x(6x) + (-1)x^2$ $\qquad [T(x^3)]_{B'} = \begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix}$

  $= 0\cdot 1 + 0\cdot x + 50x^2$

$$\boxed{[T]_{B,B'} = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 1 & -1 & 1 & 50 \end{bmatrix}_{3\times 4}} \qquad [p]_B = \begin{bmatrix} 2 \\ 8 \\ -5 \\ 4 \end{bmatrix}_B \; 4\times 1$$

$$T(p(x)) = [T]_{B,B'} * [p]_B$$

$$= \begin{bmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 1 & -1 & 1 & 50 \end{bmatrix} \begin{bmatrix} 2 \\ 8 \\ -5 \\ 4 \end{bmatrix} = \begin{bmatrix} 18 \\ -100 \\ 189 \end{bmatrix}_{3\times 1} \; B'$$

$$= 18 \cdot 1 + (-100)\cdot x + 189 x^2$$

Monday, 7_22_2019

<u>Ex</u> $V = \text{Span}(\{x^2e^{4x}, xe^{4x}, e^{4x}\})$, $B = \{\underbrace{x^2e^{4x}}_{\vec{v}_1}, \underbrace{xe^{4x}}_{\vec{v}_2}, \underbrace{e^{4x}}_{\vec{v}_3}\}$

Previously: $\boxed{D : V \to V}$ differentiation operation

$$f \mapsto D(f) = f'$$

<u>Goal</u> $[D]_B = [D]_{B,B}$

- $D(x^2e^{4x}) = [x^2 \cdot e^{4x}]' = 2xe^{4x} + 4x^2 \cdot e^{4x} = 4\vec{v}_1 + 2\vec{v}_2 + 0\vec{v}_3$

$$\langle D(x^2e^{4x})\rangle_B = \langle 4, 2, 0\rangle_B = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}_B$$

- $\mathcal{D}(xe^{4x}) = 1\cdot e^{4x} + 4xe^{4x} = 0\vec{v}_1 + 4\vec{v}_2 + 1\vec{v}_3$

$$\langle \mathcal{D}(xe^{4x})\rangle_B = \langle 0, 4, 1\rangle_B = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}_B$$

- $\mathcal{D}(e^{4x}) = 4e^{4x} = \langle 0, 0, 4\rangle_B = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}_B$

so
$$\boxed{[\mathcal{D}]_B = \begin{bmatrix} 4 & 0 & 0 \\ 2 & 4 & 0 \\ 0 & 1 & 4 \end{bmatrix}_B}$$

- Goal: Given some $f \in V$: say: $f(x) = -2x^2e^{4x} + 13xe^{4x} - 4e^{4x}$

encode
$$\boxed{[f]_B = \begin{bmatrix} -2 \\ 13 \\ -4 \end{bmatrix}_B}$$

multiply
$$[\mathcal{D}(f)]_B = [f']_B = \begin{bmatrix} 4 & 0 & 0 \\ 2 & 4 & 0 \\ 0 & 1 & 4 \end{bmatrix}_B * \begin{bmatrix} -2 \\ 13 \\ -4 \end{bmatrix}_B = \begin{bmatrix} -8 \\ 48 \\ -3 \end{bmatrix}_B$$

decode
$$\boxed{\mathcal{D}(f) = f' = -8x^2e^{4x} + 48xe^{4x} - 3e^{4x}}$$

so fast!!!