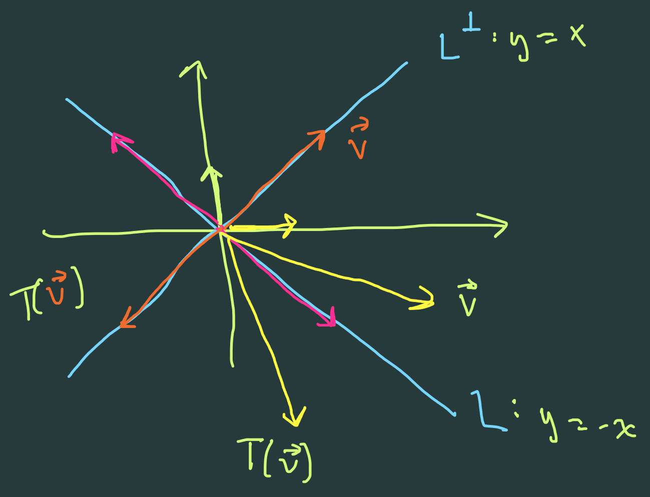
# 6.2 Geometry of Eigentheory and Computational Techniques




- $T: \mathbb{R}^2 \to \mathbb{R}^2$ is reflection across $L$
- if $\vec{v} \in L$ : $T(\vec{v}) = \vec{v}$

  $\Rightarrow$ eigenvalue is $1$

  $\vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

  $Eig(T, 1) = Span\left(\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}\right)$.

- $\vec{v} \in L^{\perp}$ : $T(\vec{v}) = -\vec{v}$

  $\Rightarrow$ eigenvalue is $-1$

  $Eig(T, -1) = Span\left(\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}\right)$.

$$\boxed{[T]_{B,B'} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}$$ ← what basis do we need to make this?

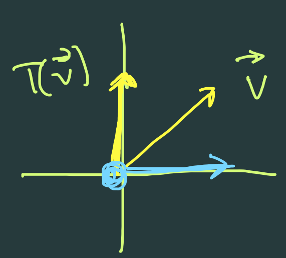
$T(\hat{i}) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = -\hat{j} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$T(\hat{j}) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = -\hat{i} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$[T] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$T(\langle x, y \rangle) = \langle -y, -x \rangle$

Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$, $\boxed{proj_y = T}$




- $\vec{v} \in$ y-axis : $T(\vec{v}) = \vec{v} \longrightarrow \lambda = 1$
- $\vec{v} \in$ x-axis : $T(\vec{v}) = \vec{0} \longrightarrow \lambda = 0$

- $Eig(T, 1) = Span\left(\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}\right)$.
- $Eig(T, 0) = Span\left(\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}\right)$.

$$\boxed{[T]_{B,B'} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}$$

$[T] = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad T(\langle x, y \rangle) = \langle 0, y \rangle$

$T(\hat{i}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad T(\hat{j}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

## • Kernel & Eigenvalues

$\lambda = 0$ is allowed! $\boxed{\text{iff}}$ $A\vec{v} = \vec{0}$ $\boxed{\text{iff}}$ $\vec{v} \in \ker(A)$
is eigenvalue — non-trivial sol — is an eigenvector $(\vec{v} \neq \vec{0})$

$\boxed{\text{iff}}$ $A$ is **not** 1-1

$\boxed{\text{iff}}$ $A$ is **not** invertible

**Theorem** Addendum to Really Big Theorem on invertibility:

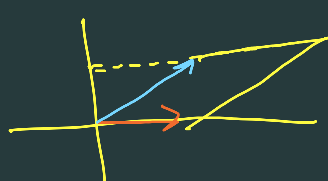
**TFAE:**
1. $A$ is invertible
2. $\lambda = 0$ is **not** an eigenvalue!
3. $\det(A) \neq 0$

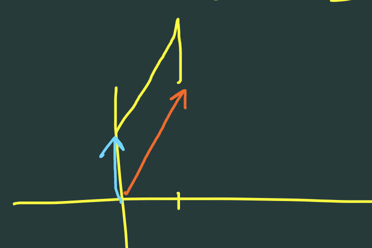
**Scaling Operator** $\boxed{S_k(\vec{v}) = k\vec{v}}$

$S_k : \mathbb{R}^n \to \mathbb{R}^n$

- $k$ is an eigenvalue of $S_k$
- $Eig(S_k, k) = \mathbb{R}^n$!

**Shear Operators** $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$





$Eig(A, 1) = Span\left(\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}\right)$ $Eig(B, 1) = Span\left(\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}\right)$

(check these are correct)