Chapter 2 Organizing and Summarizing Data

- Relative frequency $= \dfrac{\text{frequency}}{\text{sum of all frequencies}}$
- Class midpoint: The sum of consecutive lower class limits divided by 2.

Chapter 3 Numerically Summarizing Data

- Population Mean: $\mu = \dfrac{\Sigma x_i}{N}$
- Sample Mean: $\bar{x} = \dfrac{\Sigma x_i}{n}$
- Range = Largest Data Value − Smallest Data Value
- Population Standard Deviation:

$$\sigma = \sqrt{\frac{\Sigma(x_i - \mu)^2}{N}} = \sqrt{\frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}}{N}}$$

- Sample Standard Deviation

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1}}$$

- Population Variance: σ^2
- Sample Variance: s^2
- ***Empirical Rule:*** If the shape of the distribution is bell-shaped, then
  - Approximately 68% of the data lie within 1 standard deviation of the mean
  - Approximately 95% of the data lie within 2 standard deviations of the mean
  - Approximately 99.7% of the data lie within 3 standard deviations of the mean
- Population Mean from Grouped Data: $\mu = \dfrac{\Sigma x_i f_i}{\Sigma f_i}$
- Sample Mean from Grouped Data: $\bar{x} = \dfrac{\Sigma x_i f_i}{\Sigma f_i}$
- Weighted Mean: $\bar{x}_w = \dfrac{\Sigma w_i x_i}{\Sigma w_i}$
- Population Standard Deviation from Grouped Data:

$$\sigma = \sqrt{\frac{\Sigma(x_i - \mu)^2 f_i}{\Sigma f_i}} = \sqrt{\frac{\Sigma x_i^2 f_i - \frac{(\Sigma x_i f_i)^2}{\Sigma f_i}}{\Sigma f_i}}$$

- Sample Standard Deviation from Grouped Data:

$$s = \sqrt{\frac{\Sigma(x_i - \mu)^2 f_i}{(\Sigma f_i) - 1}} = \sqrt{\frac{\Sigma x_i^2 f_i - \frac{(\Sigma x_i f_i)^2}{\Sigma f_i}}{\Sigma f_i - 1}}$$

- Population z-score: $z = \dfrac{x - \mu}{\sigma}$
- Sample z-score: $z = \dfrac{x - \bar{x}}{s}$
- Interquartile Range: $\text{IQR} = Q_3 - Q_1$
- Lower and Upper Fences: Lower fence $= Q_1 - 1.5(\text{IQR})$
Upper fence $= Q_3 + 1.5(\text{IQR})$
- Five-Number Summary

Minimum, Q_1, M, Q_3, Maximum

Chapter 4 Describing the Relation between Two Variables

- Correlation Coefficient: $r = \dfrac{\Sigma\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)}{n-1}$
- The equation of the least-squares regression line is

  $\hat{y} = b_1 x + b_0$, where $\hat{y}$ is the predicted value, $b_1 = r \cdot \dfrac{s_y}{s_x}$ is the slope, and $b_0 = \bar{y} - b_1\bar{x}$ is the intercept.
- Residual = observed y − predicted y = $y - \hat{y}$
- $R^2 = r^2$ for the least-squares regression model $\hat{y} = b_1 x + b_0$
- The coefficient of determination, R^2, measures the proportion of total variation in the response variable that is explained by the least-squares regression line.

Chapter 5 Probability

- Empirical Probability

$$P(E) \approx \frac{\text{frequency of } E}{\text{number of trials of experiment}}$$

- Classical Probability

$$P(E) = \frac{\text{number of ways that } E \text{ can occur}}{\text{number of possible outcomes}} = \frac{N(E)}{N(S)}$$

- Addition Rule for Disjoint Events

$$P(E \text{ or } F) = P(E) + P(F)$$

- Addition Rule for n Disjoint Events

$$P(E \text{ or } F \text{ or } G \text{ or} \cdots) = P(E) + P(F) + P(G) + \cdots$$

- General Addition Rule

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

- Complement Rule

$$P(E^c) = 1 - P(E)$$

- Multiplication Rule for Independent Events

$$P(E \text{ and } F) = P(E) \cdot P(F)$$

- Multiplication Rule for n Independent Events

$$P(E \text{ and } F \text{ and } G \cdots) = P(E) \cdot P(F) \cdot P(G) \cdot \cdots$$

- Conditional Probability Rule

$$P(F|E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{N(E \text{ and } F)}{N(E)}$$

- General Multiplication Rule

$$P(E \text{ and } F) = P(E) \cdot P(F|E)$$

- Factorial

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 3 \cdot 2 \cdot 1$$

- Permutation of n objects taken r at a time:

$$_nP_r = \frac{n!}{(n-r)!}$$

- Combination of n objects taken r at a time:

$$_nC_r = \frac{n!}{r!(n-r)!}$$

- Permutations with Repetition:

$$\frac{n!}{n_1! \cdot n_2! \cdot \cdots \cdot n_k!}$$

Chapter 6 Discrete Probability Distributions

- Mean (Expected Value) of a Discrete Random Variable

$$\mu_X = \Sigma x \cdot P(x)$$

- Standard Deviation of a Discrete Random Variable

$$\sigma_X = \sqrt{\Sigma(x-\mu)^2 \cdot P(x)} = \sqrt{\Sigma[x^2 P(x)] - \mu_X^2}$$

- Binomial Probability Distribution Function

$$P(x) = {_nC_x} p^x (1-p)^{n-x}$$

- Mean and Standard Deviation of a Binomial Random Variable

$$\mu_X = np \qquad \sigma_X = \sqrt{np(1-p)}$$

- Poisson Probability Distribution Function

$$P(x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t} \quad x = 0, 1, 2, \ldots$$

- Mean and Standard Deviation of a Poisson Random Variable

$$\mu_X = \lambda t \quad \sigma_X = \sqrt{\lambda t}$$

Chapter 7 The Normal Distribution

- Standardizing a Normal Random Variable

$$z = \frac{x - \mu}{\sigma}$$

- Finding the Score: $x = \mu + z\sigma$

Chapter 8 Sampling Distributions

- Mean and Standard Deviation of the Sampling Distribution of $\bar{x}$

$$\mu_{\bar{x}} = \mu \quad \text{and} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- Sample Proportion: $\hat{p} = \dfrac{x}{n}$

- Mean and Standard Deviation of the Sampling Distribution of $\hat{p}$

$$\mu_{\hat{p}} = p \text{ and } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Chapter 9 Estimating the Value of a Parameter

Confidence Intervals

- A $(1-\alpha) \cdot 100\%$ confidence interval about p is

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- A $(1-\alpha) \cdot 100\%$ confidence interval about μ is

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

  Note: $t_{\alpha/2}$ is computed using $n-1$ degrees of freedom.

- A $(1-\alpha) \cdot 100\%$ confidence interval about σ is

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}$$

Sample Size

- To estimate the population proportion with a margin of error E at a $(1-\alpha) \cdot 100\%$ level of confidence: $n = \hat{p}(1-\hat{p})\left(\dfrac{z_{\alpha/2}}{E}\right)^2$ rounded up to the next integer, where $\hat{p}$ is a prior estimate of the population proportion, or $n = 0.25\left(\dfrac{z_{\alpha/2}}{E}\right)^2$ rounded up to the next integer when no prior estimate of p is available.
- To estimate the population mean with a margin of error E at a $(1-\alpha) \cdot 100\%$ level of confidence: $n = \left(\dfrac{z_{\alpha/2} \cdot s}{E}\right)^2$ rounded up to the next integer.

Chapter 10 Hypothesis Tests Regarding a Parameter

Test Statistics

- $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$
- $t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
- $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

Chapter 11 Inferences on Two Samples

- Test Statistic Comparing Two Population Proportions (Independent Samples)

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\hat{p}(1-\hat{p})}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} \quad \text{where} \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

- Confidence Interval for the Difference of Two Proportions (Independent Samples)

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

- Test Statistic for Matched-Pairs Data

$$t_0 = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$$

where $\bar{d}$ is the mean and s_d is the standard deviation of the differenced data.

- Confidence Interval for Matched-Pairs Data

$$\bar{d} \pm t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}}$$

Note: $t_{\alpha/2}$ is found using $n - 1$ degrees of freedom.

- Test Statistic Comparing Two Means (Independent Sampling)

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

- Confidence Interval for the Difference of Two Means (Independent Samples)

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Note: $t_{\alpha/2}$ is found using the smaller of $n_1 - 1$ or $n_2 - 1$ degrees of freedom.

- Test Statistic for Comparing Two Population Standard Deviations

$$F_0 = \frac{s_1^2}{s_2^2}$$

- Finding a Critical F for the Left Tail

$$F_{1-\alpha, n_1-1, n_2-1} = \frac{1}{F_{\alpha, n_2-1, n_1-1}}$$

Chapter 12 Inference on Categorical Data

- Expected Counts (when testing for goodness of fit)

$$E_i = \mu_i = np_i \quad \text{for} \quad i = 1, 2, \ldots, k$$

- Expected Frequencies (when testing for independence or homogeneity of proportions)

$$\text{Expected frequency} = \frac{(\text{row total})(\text{column total})}{\text{table total}}$$

- Chi-Square Test Statistic

$$\chi_0^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}} = \sum \frac{(O_i - E_i)^2}{E_i}$$

$i = 1, 2, \ldots, k$

All $E_i \geq 1$ and no more than 20% less than 5.

- Test Statistic for Comparing Two Proportions (Dependent Samples)

$$\chi_0^2 = \frac{(f_{12} - f_{21})^2}{f_{12} + f_{21}}$$

Chapter 13 Comparing Three or More Means

- Test Statistic for One-Way ANOVA

$$F = \frac{\text{Mean square due to treatment}}{\text{Mean square due to error}} = \frac{\text{MST}}{\text{MSE}}$$

where

$$\text{MST} = \frac{n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + \cdots + n_k(\bar{x}_k - \bar{x})^2}{k-1}$$

$$\text{MSE} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \cdots + (n_k - 1)s_k^2}{n - k}$$

- Test Statistic for Tukey's Test after One-Way ANOVA

$$q = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)}{\sqrt{\dfrac{s^2}{2} \cdot \left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\dfrac{s^2}{2} \cdot \left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

Chapter 14 Inference on the Least-Squares Regression Model and Multiple Regression

- Standard Error of the Estimate

$$s_e = \sqrt{\frac{\Sigma(y_i - \hat{y}_i)^2}{n-2}} = \sqrt{\frac{\Sigma \text{ residuals}^2}{n-2}}$$

- Standard error of b_1

$$s_{b_1} = \frac{s_e}{\sqrt{\Sigma(x_i - \bar{x})^2}}$$

- Test Statistic for the Slope of the Least-Squares Regression Line

$$t_0 = \frac{b_1 - \beta_1}{s_e \Big/ \sqrt{\Sigma(x_i - \bar{x})^2}} = \frac{b_1 - \beta_1}{s_{b_1}}$$

- Confidence Interval for the Slope of the Regression Line

$$b_1 \pm t_{\alpha/2} \cdot \frac{s_e}{\sqrt{\Sigma(x_i - \bar{x})^2}}$$

where $t_{\alpha/2}$ is computed with $n - 2$ degrees of freedom.

- Confidence Interval about the Mean Response of y, $\hat{y}$

$$\hat{y} \pm t_{\alpha/2} \cdot s_e \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}}$$

where x^* is the given value of the explanatory variable and $t_{\alpha/2}$ is the critical value with $n - 2$ degrees of freedom.

- Prediction Interval about an Individual Response, $\hat{y}$

$$\hat{y} \pm t_{\alpha/2} \cdot s_e \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}}$$

where x^* is the given value of the explanatory variable and $t_{\alpha/2}$ is the critical value with $n - 2$ degrees of freedom.

Chapter 15 Nonparametric Statistics

- Test Statistic for a Runs Test for Randomness

Small-Sample Case If $n_1 \le 20$ and $n_2 \le 20$, the test statistic in the runs test for randomness is r, the number of runs.
Large-Sample Case If $n_1 > 20$ or $n_2 > 20$, the test statistic is $z_0 = \frac{r - \mu_r}{\sigma_r}$ where

$$\mu_r = \frac{2n_1 n_2}{n} + 1 \quad \text{and} \quad \sigma_r = \sqrt{\frac{2n_1 n_2(2n_1 n_2 - n)}{n^2(n-1)}}$$

- Test Statistic for a One-Sample Sign Test

Small-Sample Case ($n \le 25$)

Two-Tailed	Left-Tailed	Right-Tailed
$H_0: M = M_0$	$H_0: M = M_0$	$H_0: M = M_0$
$H_1: M \ne M_0$	$H_1: M < M_0$	$H_1: M > M_0$
The test statistic, k, is the smaller of the number of minus signs or plus signs.	The test statistic, k, is the number of plus signs.	The test statistic, k, is the number of minus signs.

Large-Sample Case ($n > 25$) The test statistic, z_0, is

$$z_0 = \frac{(k + 0.5) - \frac{n}{2}}{\frac{\sqrt{n}}{2}}$$

where n is the number of minus and plus signs and k is obtained as described in the small sample case.

- Test Statistic for the Wilcoxon Matched-Pairs Signed-Ranks Test

Small-Sample Case ($n \le 30$)

Two-Tailed	Left-Tailed	Right-Tailed
$H_0: M_D = 0$	$H_0: M_D = 0$	$H_0: M_D = 0$
$H_1: M_D \ne 0$	$H_1: M_D < 0$	$H_0: M_D > 0$
Test Statistic: T is the smaller of T_+ or T_-	**Test Statistic:** $T = T_+$	**Test Statistic:** $T = \lvert T_- \rvert$

Large-Sample Case ($n > 30$)

$$z_0 = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

where T is the test statistic from the small-sample case.

- Test Statistic for the Mann–Whitney Test

Small-Sample Case ($n_1 \le 20$ and $n_2 \le 20$)
If S is the sum of the ranks corresponding to the sample from population X, then the test statistic, T, is given by

$$T = S - \frac{n_1(n_1 + 1)}{2}$$

Note: The value of S is always obtained by summing the ranks of the sample data that correspond to M_X in the hypothesis.
Large-Sample Case ($n_1 > 20$) or ($n_2 > 20$)

$$z_0 = \frac{T - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2(n_1 + n_2 + 1)}{12}}}$$

- Test Statistic for Spearman's Rank Correlation Test

$$r_s = 1 - \frac{6\Sigma d_i^2}{n(n^2 - 1)}$$

where $d_i =$ the difference in the ranks of the two observations in the i^{th} ordered pair.

- Test Statistic for the Kruskal–Wallis Test

$$H = \frac{12}{N(N+1)} \sum \frac{1}{n_i}\left[R_i - \frac{n_i(N+1)}{2}\right]^2$$
$$= \frac{12}{N(N+1)}\left[\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \cdots + \frac{R_k^2}{n_k}\right] - 3(N+1)$$

where R_i is the sum of the ranks in the ith sample.

Table I

Random Numbers

Row Number	Column Number 01–05	06–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
01	89392	23212	74483	36590	25956	36544	68518	40805	09980	00467
02	61458	17639	96252	95649	73727	33912	72896	66218	52341	97141
03	11452	74197	81962	48443	90360	26480	73231	37740	26628	44690
04	27575	04429	31308	02241	01698	19191	18948	78871	36030	23980
05	36829	59109	88976	46845	28329	47460	88944	08264	00843	84592
06	81902	93458	42161	26099	09419	89073	82849	09160	61845	40906
07	59761	55212	33360	68751	86737	79743	85262	31887	37879	17525
08	46827	25906	64708	20307	78423	15910	86548	08763	47050	18513
09	24040	66449	32353	83668	13874	86741	81312	54185	78824	00718
10	98144	96372	50277	15571	82261	66628	31457	00377	63423	55141
11	14228	17930	30118	00438	49666	65189	62869	31304	17117	71489
12	55366	51057	90065	14791	62426	02957	85518	28822	30588	32798
13	96101	30646	35526	90389	73634	79304	96635	06626	94683	16696
14	38152	55474	30153	26525	83647	31988	82182	98377	33802	80471
15	85007	18416	24661	95581	45868	15662	28906	36392	07617	50248
16	85544	15890	80011	18160	33468	84106	40603	01315	74664	20553
17	10446	20699	98370	17684	16932	80449	92654	02084	19985	59321
18	67237	45509	17638	65115	29757	80705	82686	48565	72612	61760
19	23026	89817	05403	82209	30573	47501	00135	33955	50250	72592
20	67411	58542	18678	46491	13219	84084	27783	34508	55158	78742

Table II

Critical Values (CV) for Correlation Coefficient

n	CV	*n*	CV	*n*	CV	*n*	CV
3	0.997	10	0.632	17	0.482	24	0.404
4	0.950	11	0.602	18	0.468	25	0.396
5	0.878	12	0.576	19	0.456	26	0.388
6	0.811	13	0.553	20	0.444	27	0.381
7	0.754	14	0.532	21	0.433	28	0.374
8	0.707	15	0.514	22	0.423	29	0.367
9	0.666	16	0.497	23	0.413	30	0.361

Table VI

Critical Values for Normal Probability Plots

Sample Size, *n*	Critical Value	Sample Size, *n*	Critical Value	Sample Size, *n*	Critical Value
5	0.880	13	0.932	21	0.952
6	0.888	14	0.935	22	0.954
7	0.898	15	0.939	23	0.956
8	0.906	16	0.941	24	0.957
9	0.912	17	0.944	25	0.959
10	0.918	18	0.946	30	0.960
11	0.923	19	0.949		
12	0.928	20	0.951		

Source: S. W. Looney and T. R. Gulledge, Jr. "Use of the Correlation Coefficient with Normal Probability Plots," *American Statistician* 39(Feb. 1985): 75–79.

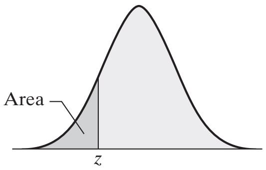


Table V

Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Confidence Interval Critical Values, $z_{\alpha/2}$

Level of Confidence	Critical Value, $z_{\alpha/2}$
0.90 or 90%	1.645
0.95 or 95%	1.96
0.98 or 98%	2.33
0.99 or 99%	2.575

Hypothesis Testing Critical Values

Level of Significance, α	Left-Tailed	Right-Tailed	Two-Tailed
0.10	−1.28	1.28	±1.645
0.05	−1.645	1.645	±1.96
0.01	−2.33	2.33	±2.575

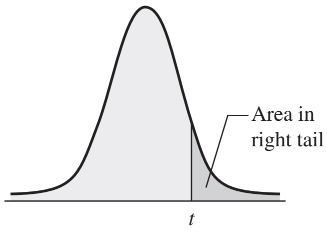


Table VII

t-Distribution
Area in Right Tail

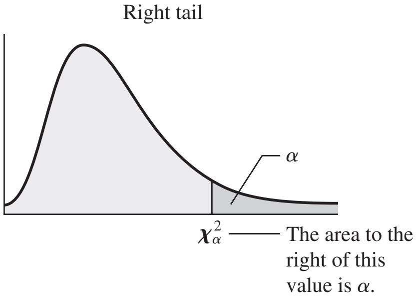
df	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.894	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
31	0.682	0.853	1.054	1.309	1.696	2.040	2.144	2.453	2.744	3.022	3.375	3.633
32	0.682	0.853	1.054	1.309	1.694	2.037	2.141	2.449	2.738	3.015	3.365	3.622
33	0.682	0.853	1.053	1.308	1.692	2.035	2.138	2.445	2.733	3.008	3.356	3.611
34	0.682	0.852	1.052	1.307	1.691	2.032	2.136	2.441	2.728	3.002	3.348	3.601
35	0.682	0.852	1.052	1.306	1.690	2.030	2.133	2.438	2.724	2.996	3.340	3.591
36	0.681	0.852	1.052	1.306	1.688	2.028	2.131	2.434	2.719	2.990	3.333	3.582
37	0.681	0.851	1.051	1.305	1.687	2.026	2.129	2.431	2.715	2.985	3.326	3.574
38	0.681	0.851	1.051	1.304	1.686	2.024	2.127	2.429	2.712	2.980	3.319	3.566
39	0.681	0.851	1.050	1.304	1.685	2.023	2.125	2.426	2.708	2.976	3.313	3.558
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
70	0.678	0.847	1.044	1.294	1.667	1.994	2.093	2.381	2.648	2.899	3.211	3.435
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
90	0.677	0.846	1.042	1.291	1.662	1.987	2.084	2.368	2.632	2.878	3.183	3.402
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291

Table VIII

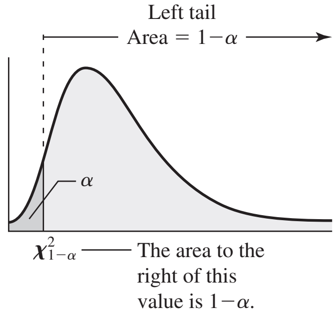
Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of Critical Value									
	0.995	**0.99**	**0.975**	**0.95**	**0.90**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Right tail



Left tail



Two tails