What is Calculus?

A Brief History of Math and Calculus

January 24, 2018

Math 30 - Calculus I

Pitzer College

Professor: Dr. Jorge Eduardo Basilio

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Ancient Problem 3*: What is a (real) Number? (Egypt, Mesopotamia, Greek)

Ancient Problem 4: Tangent Line Problem (Greek, Europe Middle Ages)

Cultural Context

Timeline of ancient cultures

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- Also: India and China has ancient cultures which parallel much of the above

Want to know more?

Calculus Gems: Brief Lives and Memorable Mathematics

by George F. Simmons (Author)

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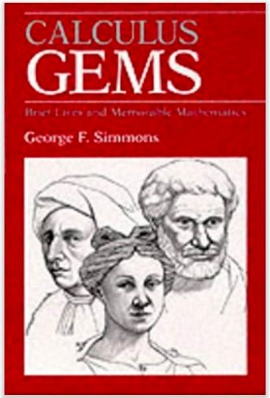
Look inside



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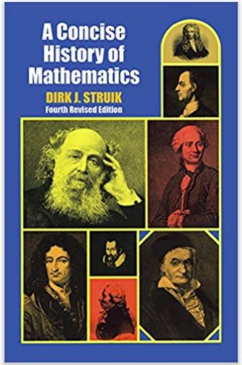

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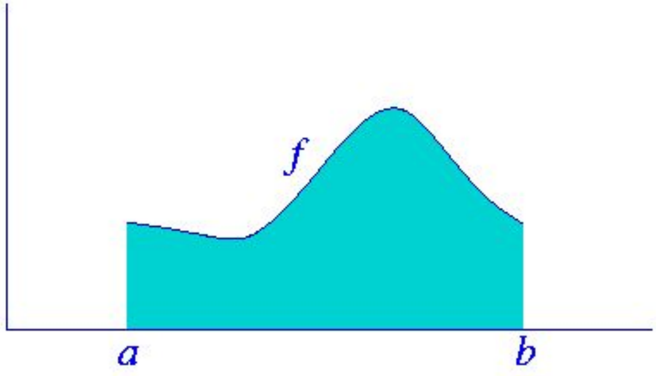
Ancient Problem 1: Area under a curve (Egypt, Mesopotamia, Greek)

In Egypt, taxes were collected according to how much land you had.

Because the Nile river floods, the amount of land you had would change through the seasons. Thus, it was necessary to figure out the area of your land according to a boundary shaped by a complex curve.



Notice: you can't use simple geometric shapes (rectangles, triangles, circles) to calculate such an area exactly

AP1: What is the area under a curve?

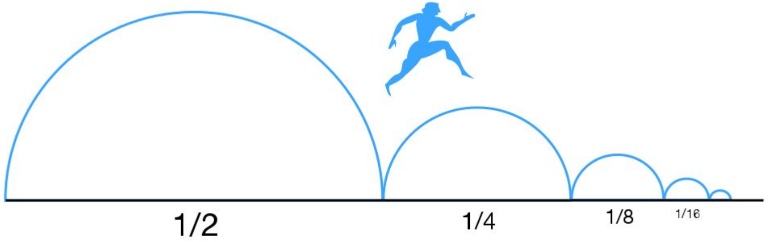
Ancient Problem 2: Problem of Motion ie instantaneous velocity (Greek)

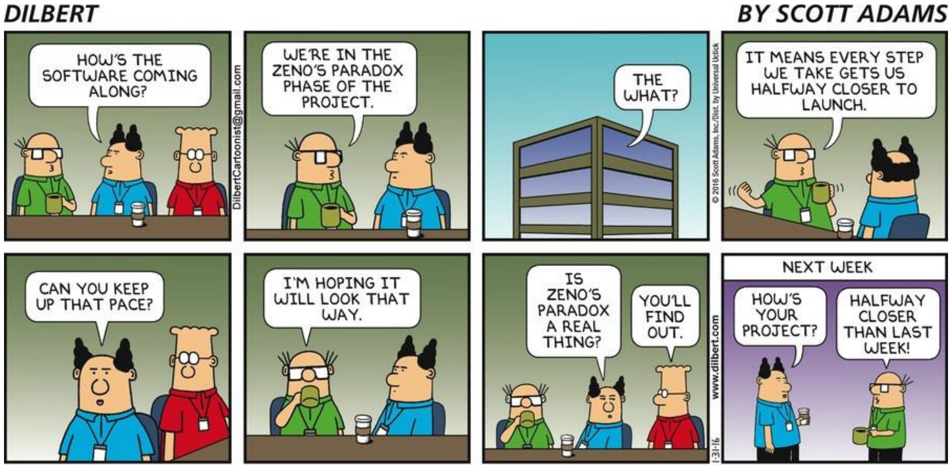
Zeno’s Paradox: Motion is impossible

How does one define motion?

Velocity = Distance / time

Velocity at an instant?? Divide by 0?

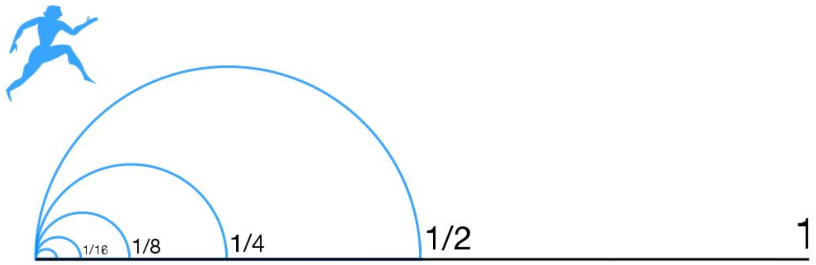




AP2: What is instantaneous velocity?

Ancient Problem 2: Problem of Motion ie instantaneous velocity (Greek)

Zeno of Elea (c. 490-430 BCE)



There are three main paradoxes attributed to Zeno extant from Aristotle's *Physics:*

Achilles and the Tortoise, The Dichotomy, The Arrow

The Dichotomy:

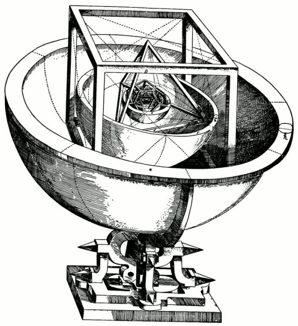
> That which is in locomotion must arrive at the half-way stage before it arrives at the goal.
>
> —as recounted by Aristotle, *Physics*

Ancient Problem 3: What is a (real) number? (Egypt, Mesopotamia, Greek)

- Babylonians: invented very complex number system that included decimals called the **positional notation**

Ex: 12.65 = 1*10+2*1 + 6*(1/10)+5*(1/100)

Understood irrational numbers like sqrt(2)

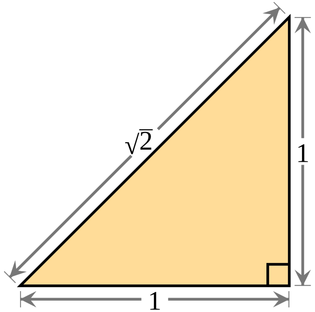


- Greeks: Pythagoras: **“all is number”**

Pi, sqrt(2), were not numbers

Only considered fractions to be numbers and irrational numbers were ‘magnitudes’

Because of this: geometry and numbers developed separately



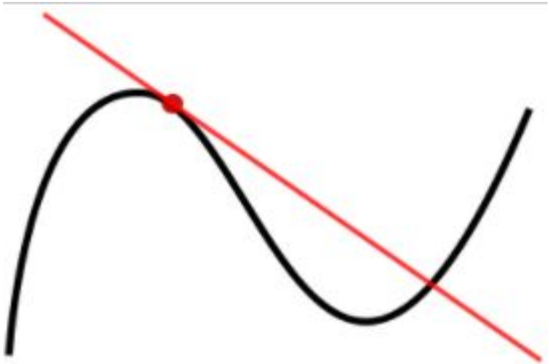
Ancient Problem 4: Tangent Line Problem (Greek, Europe Renaissance)

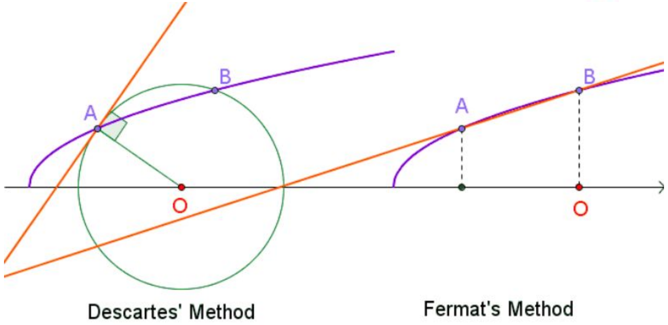
This one is clearly more modern.

AP4: Given a curve and a point P on the curve, find the line that also passes through P that is "infinitely close to the curve" (Liebniz)

Tangent lines were important in Greek geometry. Archimedes (~300 BCE) found the tangent line to a spiral (first to find a tangent to a non-circular curve)

Many Renaissance mathematicians: Descartes, Fermat, & more solved it for specific functions like polynomials of low degrees





In both cases, O slides along horizontal axis until A=B.

How were these problems solved?

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- *Open Stewart’s A Preview of Calculus*
    - *AP1: Area of a circle via the “Method of Exhaustion”*
    - *AP4: Tangent Line Problem*
    - *AP2: Instantaneous Velocity*
    - *Limit of a sequence (related to Zeno’s Paradox/AP2 and also irrational numbers/AP3)*
    - *Sum of a series (related to irrational numbers/AP3)*

Many different questions, “same” approach towards a solution -- But must deal with limits/infinity

A bit more history...

Ingredients for Calculus?

The Greeks developed planar geometry to an impressive degree of sophistication and certainly were struggling with APs 1-4 mainly from a philosophical point of view and not necessarily concerned with applications to the real world.

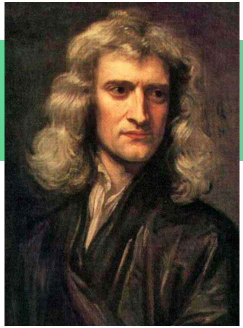
What kept them from developing the calculus?

In short, notation. The modern notation of numbers was not yet invented. Babylonian system of numbers was thousands of years older than the Greek but it was not pursued by them in favor of the whole numbers due to mystic reasons (thanks Pythagoras!). It was hundreds of years after the Greeks when the indian/islamic mathematicians adapted the decimal notation from the Babylonians with a nicer way of writing numbers using the symbols 0,1,2,...,9, which you now recognize. But "number" itself is a tool and not the impetus for applications. These came from algebra equations. The indian/islamic mathematicians developed the theory of algebra (briefly, solving polynomial equations) with the notation of today. This helped pave the way for the quintessential invention needed for calculus--the function.

Functions

Essentially encode "variation." variation is code for change. Compare "static" equation $2x+y=3$ with the dynamic function $f(x)=3-2x$. They are the same thing but the function point of view implies variation, dynamics, input/output. By the way, the function concept was a tricky thing. It was developed hundreds of years AFTER Newton/Leibniz invented calculus.

A bit more history...



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Wait! I thought Newton/Leibniz invented calculus!

Sort of, main reason for their fame: they are the first to observe and prove that AP1 and AP4 are related problems, they are 'inverse to each other' or:

AP1 & AP4 are TWO SIDES OF THE SAME COIN

Besides solving the APs, the subject of calculus is unparalleled at solving hard problems. Two main branches of calculus:

1. AP1 (Area Problem) became known as integral calculus
2. AP4 (Tangent Line Problem) because known as the differential calculus

Fear of Infinity

Infinity has captivated mathematicians, philosophers, and poets like no other concept. It is as alluring as it is wicked.

The rules for how to correctly work with the concept are not obvious (unlike the rules for Euclidean geometry).

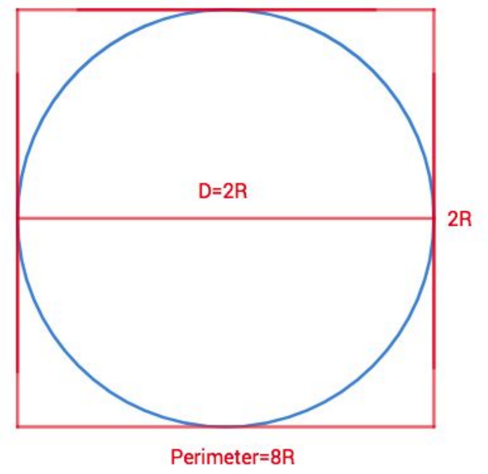
It took a long time to understand what is allowed and what is not allowed in infinite processes. The result of these studies is calculus, which you will now learn.

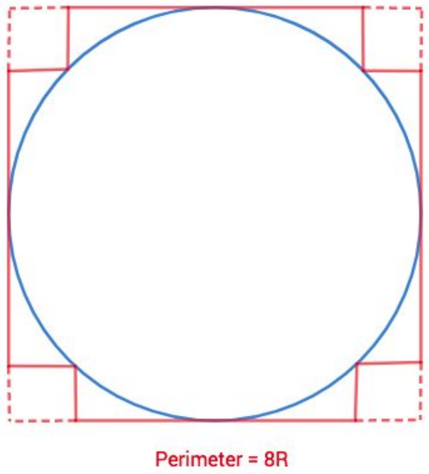
Here's two examples of showing what can go wrong:

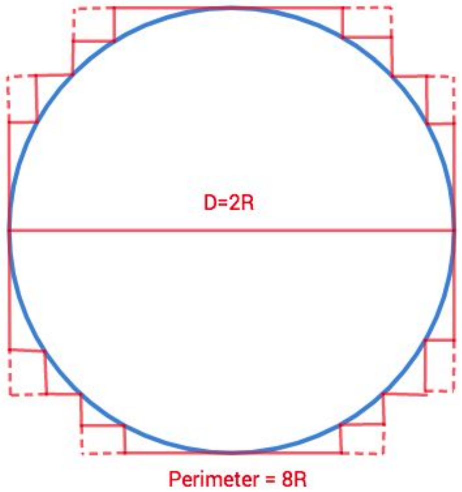
- Zeno's Paradoxes stumped the best minds for thousands of years
- $\pi = 4$

Does π = 4?

Start with a circle of radius R. We first estimate the perimeter by 8R using a circumscribed square. Cut out four corners and use the new shorter sides to approximate the perimeter. Notice that the perimeter is still 8R. Approximate the circle with more and more with sides parallel to axes. The perimeter is still 8R. Limit of these jagged curves approximates the circle so the limit of these perimeter is the circumference of the circle. Thus, perimeter of full circle is 8R. Because C=2*π*R we have 8R=2*π*R so 4=π!







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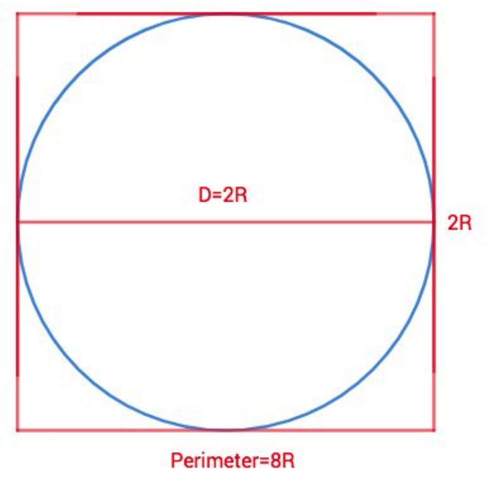
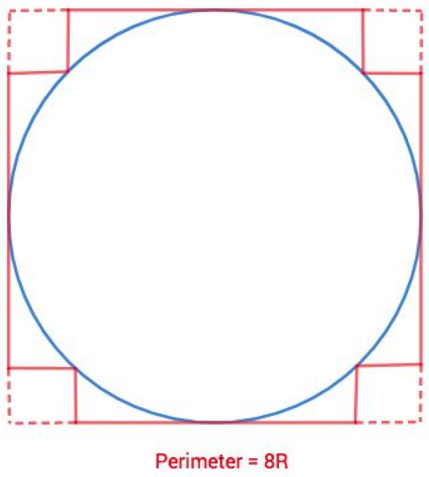
Does $\pi = 4$?

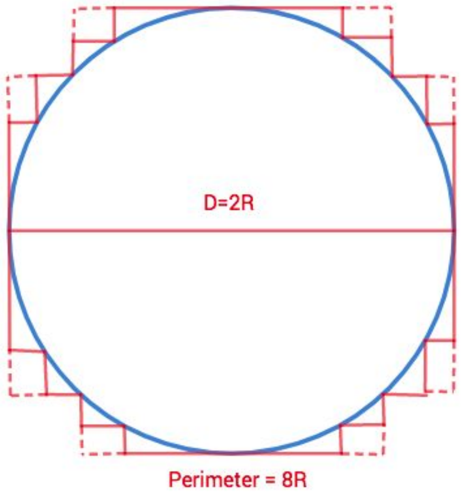
Of course not!

There is a flaw in this argument that's very difficult to catch.

Can you find it?







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