S A R A H • L A W R E N C E • C O L L E G E

HHW# 2 - Midterm #2 FALL 2016

Math 3005. Calculus 1. Jorge Eduardo Basilio

Posted on Thurs 11/30

DIRECTIONS

- All work turned in must be NEAT, ORGANIZED, AND STAPLED. Handwriting should be neat and legible.
- Also, it must have a cover page with the following information: the Course Name, Section, Instructor's name (me!), Date Due, name of the assignment and your name all neatly and clearly marked. In addition, write the following statement and sign-your name under it: *"I swear to neither receive nor give any unauthorized assistance on this assignment."*
- It must be written on blank or lined 8.5" by 11" sized paper using only the front side of the page.
- It can be written in pen or pencil. If written in pen, mistakes must be neatly crossed out, or erased with white-out.
- Be sure to label each problem carefully and staple work sequentially.
- Be sure to show all work on every problem.
- The level of "polish" should be very high, just like it would be when turning in an essay in an English class. You should work out solutions on scratch paper before you write your final draft. When applicable, write in complete sentences.
- This assignment is open book and open notes (from seminar, RPs, or previous HHWs), however, you are to work entirely on your own (or you may ask me questions). Consulting other people, other textbooks or the internet is strictly forbidden.

DUE THURSDAY, DECEMBER 8, 8:59 PM

You can slide it under my door at any time before the above deadline. Unless for extra-ordinary reasons, no late assignments will be counted/graded.

Additional Instructions

• Answers to all Problems (1-10) must be given as a complete sentence ($M \rightarrow E$).
• In addition, for Problems 2, 3, 8, 9, 10, please label and include the steps of the "EME System" in your solutions.
• In addition, for Problems 7,8,9,10, please label and include the Primary Equation (PE) and the Secondary Equation(s) (SE). If there are no (SE), then write either "none" or the domain of feasibility for the quantiles in question.
• No calculators allowed with the exception of Problem 6 where you may use it to find approximate answers to critical numbers and function values. But do not use any graphing features. In other words, please show all necessary algebra steps.

Problems

1. [10] Find the equation of the tangent line of $x^{2/3} + y^{2/3} = 4$ at the point $(-3\sqrt{3}, -1)$. (Hint: Use $3\sqrt{3} = 3 \cdot 3^{1/2} = 3^{3/2}$, but no calculator.)

2. [10] Metalhead Moe is flying his kite 100 feet above his hand at McCarren park. The kite moves away from Moe in a direction parallel to the ground at a rate of 10 feet per second. At what rate must the string be let out when the length of the string is already let out is 200 feet?

3. [10] Given that a mothball (assume a sphere) evaporates at a rate proportional to its surface area, $4\pi r^2$, show that the rate of change of the radius is constant. Is this constant positive or negative? Defend your reasoning.

4. [25] You are given a function $H(x)$ below and its derivatives:

$$H(x) = \frac{-3(x^2+16)}{x^2-16}, \qquad H'(x) = \frac{160x}{(x^2-16)^2}, \qquad H''(x) = -\frac{160(3x^2+16)}{(x^2-16)^3}.$$

Fill out a complete:

(i) CSI Line for H'
(ii) CSI Line for H''
(iii) Determine all extrema, distinguish between both local and global.
(iv) Determine the CSInfo: Domain, Intercepts, Asymptotes.
(v) Sketch the graph of $H(x)$.

You may use that the following points are on the graph of $H(x)$:

$$(0, 3), \qquad (\pm 5, -\frac{41}{3}), \qquad \left(\pm 10, -\frac{29}{7}\right),$$

with $\frac{41}{3} \approx 13.667$ and $\frac{29}{7} \approx 4.143$, but you are not allowed to use graphing technology.

5. [15] Multiple choice. However, please explain your reasoning with sound mathematics.

(i) Assume that f and g are differentiable functions and $g(x) < 0$ for all real numbers x. If $f'(x) = (x^2 - 4)g(x)$, which of the following must be TRUE?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$

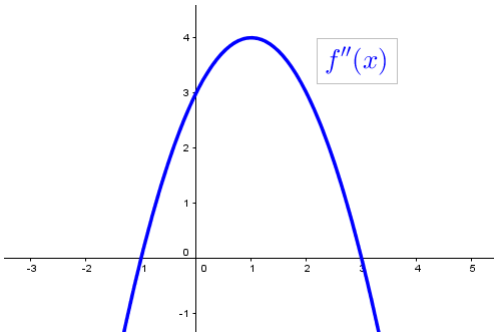
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$

(C) f has a relative minimum at $x = -2$ and at $x = 2$

(D) f has a relative maximum at $x = -2$ and at $x = 2$

(E) It cannot be determined from the information provided

(ii) The graph of $f''(x)$, the second derivative of the function f, is shown below. On what intervals is f concave UP?



(A) The graph of f is concave up for NO values of x

(B) The graph of f is concave up on the interval $(-1, 3)$

(C) The graph of f is concave up on the interval $(-\infty, +\infty)$

(D) The graph of f is concave up on the interval $(-\infty, -1) \cup (3, +\infty)$

(E) The graph of f is concave up on the interval $(3, +\infty)$

(iii) Which of the following statements is TRUE?

(A) A continuous function on (a, b) must have local extrema
(B) A continuous function on $[a, b]$ must have local extrema
(C) A continuous function on (a, b) must have global extrema
(D) A continuous function on $[a, b]$ must have global extrema
(E) None of the above.

6. [25] Let $W(x)$ be the function defined by

$$W(x) = x^2 \ln |x| + 1 - x^2.$$

(i) Find all CN1s and fill out a CSI Line for $W'(x)$.

(ii) Find all CN2s and fill out a CSI Line for $W''(x)$.

(iii) Find all extrema, distinguish between both local and global.

(iv) Sketch the graph of $W(x)$.

You may use that the intercepts of W are: $(\pm 1, 0), (\pm 2.218, 0)$. Also, you may use without proof the following limits:

$$\lim_{x \to 0} W(x) = 1 \qquad \text{and} \qquad \lim_{x \to \pm\infty} W(x) = +\infty.$$

You may use a calculator only to find numerical answers but you may not use any graphing software.

7. [10] Find the point Q on the curve $y = f(x)$ that is closest to the point $P = (12, 0)$, where $f(x) = \sqrt{x - 8}$.

8. [10] A rectangular page will contain 50 square inches of print. The margins at the top and bottom of the page are to be 2 inches wide. The margins on each side are to be 1 inch wide. Find the dimensions of the page that will minimize the amount of paper used.

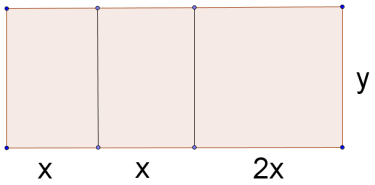
9. [10] The hottest toy of 2014 is the lovable robotic T-rex, Boomer! The monthly costs of making this great toy is given by $C(x) = \dfrac{2,000,000x}{0.03x^2 + 7500}$, where x is the number of Boomers made (in millions). Find the number of Boomers that makes the marginal cost zero (In Economics, the term "marginal" means derivative).

10. [10] Steve Job's biggest fan, Emilia, decides to start a t-shirt company named *Pomme d'Ordinateur* specializing in turtlenecks. She models the annual price function by $p = 128 - 0.5x$, where x represents thousands of turtlenecks produced and sold. The cost function is modeled by $C = 50x - 37.75$. Find the PRICE which Emilia should sell turtlenecks that will maximize her profits. (Recall that a revenue function is $R(x) = price \cdot items$ and the Profit function is $P(x) = R(x) - C(x)$).

Extra Credit

EC1. [2] Give the primary equation and the secondary equation(s) only. **Do not solve the problem completely!**

Jurrasic Park is building a new enclosure to contain the meat-eating Velociraptors on the left, the meat-eating Tyrannosaurus in the middle, and the plant-eating Stegosauruses on the right. The fencing around the meat-eating dinosaurs costs $20,000 per meter since it needs to be electrified and much stronger than the fencing which encloses the Stegasaurus area. The fencing around the Stegasauruses costs $13,000 per meter. The enclosure must be 4,000 square meters. Find the dimensions of the fencing that minimizes *costs*.



EC2. [5] You are given a continuous function for which $f''(x) > 0$ for all real numbers x, except at $x = a$. **True or False:** f might have an absolute maximum at $x = a$.

Justify your answer with examples or sketches of functions.

EC3. [10] Suppose $f''(x) < 0$ for x near a value a. Then the linearization of f at a (i.e. the tangent line at a) is

(A) an over approximation

(B) an under approximation

(C) unknown without more information

Explain your reasoning both with sketch and in words.