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Summary of Convergence and Divergence Tests for Series

TEST	SERIES	CONVERGENCE OR DIVERGENCE	COMMENTS
*n*th-term	$\sum a_n$	Diverges if $\lim\limits_{n\to\infty} a_n \neq 0$	Inconclusive if $\lim\limits_{n\to\infty} a_n = 0$
Geometric Series	$\sum_{n=1}^{\infty} ar^{n-1}$	(i) Converges with sum $S = \frac{a}{1-r}$ if $\lvert r\rvert < 1$ (ii) Diverges if $\lvert r\rvert \geq 1$	Useful for comparison tests if the nth term a_n of a series is similar to ar^{n-1}
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	(i) Converges if $p > 1$ (ii) Diverges if $p \leq 1$	Useful for comparison tests if the nth term a_n of a series is similar to ${}^1/_{n^p}$
Integral	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n)$	(i) Converges if $\int_1^\infty f(x)dx$ converges (ii) Diverges if $\int_1^\infty f(x)dx$ diverges	The function f obtained from $a_n = f(n)$ must be continuous, positive, decreasing, and readily integrable.
Comparison	$\sum a_n, \sum b_n$ $a_n > 0, b_n > 0$	(i) If $\sum b_n$ converges and $a_n \leq b_n$ for every *n*, then $\sum a_n$ converges. (ii) If $\sum b_n$ diverges and $a_n \geq b_n$ for every *n*, then $\sum a_n$ diverges. (iii) If $\lim\limits_{n\to\infty} \left({}^{a_n}/_{b_n}\right) = c > 0$, then both series converge or both diverge.	The comparison series $\sum b_n$ is often a geometric series or a p-series. To find b_n in (iii), consider only the terms of a_n that have the greatest effect on the magnitude.
Ratio	$\sum a_n$	If $\lim\limits_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L\ (or\ \infty)$, the series (i) Converges (absolutely) if $L < 1$ (ii) Diverges if $L > 1\ (or\ \infty)$	Inconclusive if $L = 1$ Useful if a_n involves factorials or n*th* powers If $a_n > 0$ for every *n*, the absolute value sign may be disregarded.
Root	$\sum a_n$	If $\lim\limits_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L\ (or\ \infty)$, the series (i) Converges (absolutely) if $L < 1$ (ii) Diverges if $L > 1\ (or\ \infty)$	Inconclusive if $L = 1$ Useful if a_n involves n*th* powers If $a_n > 0$ for every *n*, the absolute value sign may be disregarded.
Alternating Series	$\sum (-1)^n a_n$ $a_n > 0$	Converges if $a_k \geq a_{k+1}$ for every *k* and $\lim\limits_{n\to\infty} a_n = 0$	Applicable only to an alternating series
$\sum \lvert a_n\rvert$	$\sum a_n$	If $\sum \lvert a_n\rvert$ converges, then $\sum a_n$ converges.	Useful for series that contain both positive and negative terms