SARAH • LAWRENCE • COLLEGE

MATH 3135: COURSE SYLLABUS SPRING 2017

What are Numbers? A Narrative of 30,000 Years and Counting...

Level: Open Updated: 1.17.2017

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Meeting Times

Mondays & Wednesdays – 3:35 pm - 5:00 pm – Science Center – Room 201.
Individual student conferences to be scheduled on alternating weeks.

What is this class?

Have you ever wondered what constitutes numbers? Are they *real* or *imagined*? Did we discover or create them? What is infinity? These fundamental questions have been an inexhaustible source of inspiration to countless thinkers, poets and artists.

We will begin our story in 30,000 B.C., about 27,000 years before the dawn of civilization, with primitive forms of counting. We will then jump forward to the fertile crescent region to study the various forms of number systems and symbols of the ancient Sumerians, Babylonians and Greeks. We'll continue with a study of arithmetic using whole numbers (1,2,...), rational numbers (fractions) and the algebra and geometry associated to them from various viewpoints from antiquity to modernity. Along the way, we'll learn the story of "0" and why the Pythagoreans would kill to defend the secret of the discovery that $\sqrt{2}$ was not a (rational) number! We'll explore how the "real numbers" are infinitely complex and how the "complex numbers" are just as real as any other mathematical object. In the second part of the course, we'll begin a cultural study of infinity where we'll puzzle over numerous paradoxes and learn that, in fact, modern mathematicians are perfectly comfortable with the idea that there isn't just *one infinity*. Time permitting, and according to the tastes and wishes of the class, we can discuss how to logically and rigorously construct the natural numbers and prove theorems such as: 1+1=2. By the end of our course, we will have caught up to the current perspectives on the concept of numbers and learn that this narrative is far from over.

Above all, the search for an answer to "what are numbers?" is a good story about humanity. And you'll be an active participant. Not only will we learn from various sources of reading materials, but we will actively explore and "play" with various problems for ourselves and (re-) discover many gems of insight in seminar and assignments. Conference work will be allocated to clarifying course ideas and exploring additional mathematical or philosophical topics.

For conference work students may choose to undertake a deeper investigation of a single topic or application of the Calculus or conduct a study in some other branch of mathematics or a related field.

Learning Goals

To develop an understanding and appreciation of the mathematical concepts and tools in single-variable Calculus that provide the ability to model continuously changing quantities. Additionally, by the end of the semester students will be able to:

- demonstrate proficiency in the history of mathematics including important dates and milestones.
- demonstrate proficiency in treating mathematical content at an appropriate level in both symbolic (e.g. equations) and text (e.g. words) form.
- demonstrate competence in the use of numerical, graphical, and algebraic representations.
- demonstrate proficiency in the use of mathematics to formulate and solve problems.

Who should take this class?

This course is intended for students interested in the intersection of history, anthropology, literature and, of course, mathematics; as well as any student wishing to broaden and enrich the life of the mind.

Prerequisites

High-school algebra and geometry is required; high-school trigonometry is recommended. Though curiosity and desire to learn are far more important.

Required Texts

- *Number: The Language of Science*, by Tobias Dantzig
- *Mathematics in Civilization*, 3rd Edition, by H.L. Resnikoff & R.O. Wells, Jr.
- *Zero: The Biography of a Dangerous Idea*, by Charles Seife
- *Mathematics for the nonmathematician*, by Morris Kline

- *Library of Babel* by Jorge Luis Borges (distributed in class)
- *El Aleph* by Jorge Luis Borges (distributed in class)

Optional/Recommended Texts

- *The Unimaginable Mathematics of Borges' Library of Babel*, by William Goldbloom Bloch
- *Mathematics and the Imagination*, by Edward Kasner & James Newman
- & Many more suggestions can/will be made…

Expectations

Course Readings

For each seminar meeting there will be an assigned reading from the course text or a supplemental handout. As this is a lecture-free course, these readings will form the basis of our seminar discussions. In preparation for each seminar meeting, students absolutely must complete the assigned reading and be fully prepared to discuss the ideas encountered in the readings and/or work out problems alone or in groups at the blackboard. See "Course Readings and Seminar Exercises" hand-out (to be supplied on MySLC) for the scheduling of readings.

Reflection pieces

The ability to read mathematics successfully–for deep understanding and long term retention–is a skill that requires considerable effort to develop. It is also a skill that is often not developed in traditional high school courses. In this course you will have much opportunity to improve your technical reading/learning skills. As a first step, please consult the "Suggestions for Effective Reading of Mathematics" document (to be supplied on MySLC).

Students are required to prepare a summary (as little as a few sentences but sometimes they may be a paragraph or two; but no more!) and question (consult the "Suggestions for Effective Reading of Mathematics" document; to be supplied on MySLC) for each section of assigned reading in advance of each seminar meeting called "Reflection Pieces." These are due online two hours before seminar.

Assignments

In addition to completing the daily readings, students are required to prepare a collection of exercises (be it actual math problems, summaries, exploratory essays, etc.) which may be worked on alone or in groups. Due dates will be provided on MySLC. Expect an assignment due every two or three weeks.

Papers

There may be two or three additional papers, 3-5 pages long, which explore primary sources of material encountered throughout the readings. These will be announced with at least two weeks anticipation before they are due.

There are no in-class quizzes or timed exams in this course. Reflection pieces, Assignments and the Papers permit assessment of student progress in learning the topics in this course.

Conference Work

Each student in the course will be expected to design and complete an independent project for conference work. Individual conferences will be held biweekly. Student conference work may be dedicated to a deeper investigation of a single topic studied in the course, study in a different branch of mathematics (e.g., statistics, game theory, linear algebra) or some other mathematically-themed investigation (e.g. black holes). Conference time will also provide an additional, out-of-class opportunity for discussion of ideas generated in seminar.

Important dates[1]:
3/24 Friday: Conference proposal
4/21 Friday: Conference draft
5/8 Last week of class: Conference project presentation
5/19 Friday: Conference project due

Late Work Policy

Late work will not be accepted. However, two extensions will be granted to any student who requests an extension at least 24 hours in advance of the original deadline. Extension requests can be presented in person, over the phone, or via email. Except in unusual circumstances, each student will be granted only two deadline extension during the semester.

[1] Dates subject to change

Additional Help

I encourage students who are having difficulty with the course material to meet with me for individualized help. Students are also encouraged to develop and maintain an email dialogue with me so that I may provide more timely assistance with smaller-scale questions.

Evaluations

At the end of the semester an individual course evaluation and course grade will be given to each student. This evaluation will be based primarily on the students level of preparation for seminar, contributions to seminar discussion and quality of work on the problem sets, papers, and conference project. There will be no formal examinations in this course.

Self-Evaluations

Introspection is an important component of the learning process: a student's evaluation of his or her own work is as important, if not more important, than any teacher's evaluation. As such, students will be required to write a brief statement of self-evaluation on each assignment (problem sets and conference project) and an additional, more comprehensive self-evaluation at the end of the term.

Attendance

Attendance is absolutely mandatory. Students who miss more than two classes (without a documented reason) run the risk of receiving reduced course grade. Number of classes missed and number of classes with significant tardiness will be indicated on the course evaluation. If a class is missed, the student is responsible for obtaining class notes and assignments, and the student is expected to be fully prepared for the next class session.

Note: Except in cases of emergency or a full 24 hour advanced notice, there will be no rescheduling of missed conferences. However, when unavoidable situations occur, students may request an alternative conference time in advance of the regularly scheduled conference time.

Science & Math. Third Programs

Qualified students may enroll in this course as part of a Science and Mathematics third program. This registration option allows students to enroll in two science and mathematics courses that together constitute a third of the enrolling student's registration for the term. This seminar can be combined under this third program with another seminar or a lecture course and either a yearlong or semester course. Interested students should consult with the professors of both courses, and permission of both professors is required to enroll in the program.

Disability Services

If you have a disability that may interfere with your ability to participate in the activities, coursework, or assessment of the objectives of this course, you may be entitled to accommodations. Please contact Polly Waldman, Associate Dean of Studies and Disabilities Services, in Westlands 207C or x2235 or pwaldman@sarahlawrence.edu. Under the Americans with Disabilities Act and Section 504 of the Vocational Rehabilitation Act of 1973, all students, with or without disabilities, are entitled to equal access to the programs and activities of Sarah Lawrence College.

Thematic Outline

This is broad course outline of the topics covered. I will post a separate document with the entire course schedule of readings on MySLC shortly. Consult MySLC for updates.

Part 0: Pre-History & Number Sense

How far back does the story go? Can animals count? & more.

Part 1: Ancient Number Systems

We'll learn how to count and express numbers in ancient Egyptian, Babylolian, and Summarian number systems. The goal here is to be proficient enough so that if we traveled back in time to these civilizations, we'd find work as accountants ;-)

Additional systems will be explored in the assignments and/or conference work.

Part 2: Abstraction, Part I: From Counting Numbers to *Imaginary* Numbers

The Five W's of zero, negative numbers, fractions, real and complex numbers. Additional *numbers* will be explored in the assignments and/or in conference work.

Part 3: Literary Interlude

Numbers in literature. We'll read two stories from my compatriot Jorge Luis Borges' plus any additional works suggested by the class.

Part 4: Abstraction, Part II: Towers of Infinity

We'll discuss infinity in detail from multiple perspectives from mathematical, cultural, philosophical, geometric and artistic.

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Part n: Abstraction, Part III: Building Number Systems Rigorously

If time permits, and according to class wishes, we can study how to mathematically construct the counting numbers, $\{1, 2, 3, \ldots\}$, and prove some theorems.