MATH 32: Calculus III — SPRING 2018

EXAM 3 — **Dr. Basilio**

Pitzer College

Due — Monday 4.30.2018 at 5 pm

Honesty Pledge

On my honor, by printing and signing my name below, I vow to neither receive nor given any unauthorized assistance on this examination:

NAME (PRINT): ____________________ SIGNATURE: ______________________

Directions

- YOU ARE **NOT** ALLOWED TO USE A CALCULATOR ON THIS EXAM.
- Once you read this document, you are not allowed to consult any textbook, notes, other people, or the internet.
- Please print this exam and write your answers in the space provided. All work must be shown to receive credit. You may chose to do some scratch work on another page, but be sure to write any relevant calculations on your printed exam copy.
- Include a cover page (including: title, your name, course, my name, date due), write on the front side of each page only, and staple your work.
- You must work entirely on your own! No tutors, working with classmates, using the textbook, online sources, or other outside help. Students suspected of cheating will be given a zero for this exam and reported to the appropriate office.
- Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.
- Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown. *If in doubt, ask for clarification.*
- Leave answers in exact form (as simplified as possible), unless told otherwise.
- Put a $\boxed{\text{box around your final answer}}$ where applicable.
- Some questions contain multiple-parts which you must do individually and the parts are denoted by (i), (ii), (iii), etc. Some questions are multiple-choice and the choices are denoted with (A), (B), (C), (D), and (E).
- If you need extra space, there is extra space on the back of the cover page and clearly indicate that you are continuing your work there in the original location.
- **PLEASE CHECK YOUR WORK!!!**

Problem	Points	Score
1	10	
2	10	
3	30	
4	30	
5	20	
6	10	
$\sum$	110	

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Problem 1 (10 points)

TRUE or FALSE? Mark each of the following statements as “True” of “False” and be sure to spell out the entire word. Give a very brief justification for your answer.

If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

__________ (i) If $\vec{F}$ is a differentiable vector field in the plane and C is the curve parametrized by $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$ for $t \in [0, 2\pi]$, then $\oint_C \vec{F} \cdot d\vec{r} = 0$.

__________ (ii) By Fubini's Theorem: $\int_0^1 \int_{x^2}^{\sqrt{x}} xy \ln(x^2 + y^2 + 1)\, dy\, dx = \int_{x^2}^{\sqrt{x}} \int_0^1 xy \ln(x + y + 1)\, dx\, dy$

__________ (iii) If $\vec{F} = \nabla f$ for a scalar-valued function f and if C_1 and C_2 are two paths in space that start and end at the same points $A = (x_1, y_1, z_1)$ and $B = (x_2, y_2, z_2)$, then $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$

__________ (iv) The infinitesimal unit of volume in Spherical Coordinates is $dV = r\, dr\, d\theta\, dz$

__________ (v) The subset $D = \{(r, \theta) \mid 1 < r < 2, 0 \leq \theta \leq 2\pi\}$ in the plane is simply connected.

Problem 2 (10 points)

Determine the extrema of $f(x, y) = x - 2y$ subject to the constraint $x^2/4 + y^2 = 2$ using the Method of Lagrange Multipliers (no other method will receive points).

Problem 3 (30 points)

(i) Find $\displaystyle\iiint_E y\,dV$, where E is the solid region which lies under the plane $z = 2x + 3y$ and above the domain in the xy-plane bounded by the curves $y = x^2$, $y = 0$, and $x = 1$.

(ii) Consider

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} 1\,dz\,dy\,dx.$$

Sketch and identify the type of region W (corresponding to the way the integral is written by giving the inequalities as done in class). Also, explain the geometric meaning of this integral.

(iii) Evaluate the integral in part (ii) using Spherical Coordinates.

Problem 4 (30 points)

(i) Find the volume under the graph of $f(x, y) = x^3y^2$ lying above the rectangle $[1, 2] \times [0, 4]$.

(ii) Change the order of integration appropriately in order to evaluate the following integral:

$$\int_0^{16} \int_{\sqrt{x}}^{4} \sin(y^3)\, dy\, dx.$$

(iii) Evaluate using Polar Coordinates:

$$\int_{-1}^{0} \int_0^{\sqrt{1-y^2}} \frac{1}{\sqrt{4 - x^2 - y^2}}\, dxdy$$

Problem 5 (20 points)

Consider the following vector field

$$\vec{F}(x,y) = \langle 2xy + y^3, x^2 + 3xy^2 + 2y \rangle.$$

(i) Show that $\vec{F}$ is conservative.

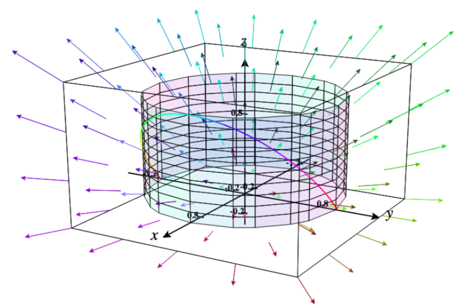
(ii) Find the potential functions f for $\vec{F}$ (i.e. $\vec{F} = \nabla f$).

(iii) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve parametrized by $\vec{r}(t) = \langle \frac{8}{9\pi\sqrt{2}} t\cos(t), \frac{8}{9\pi\sqrt{2}} t\sin(t) \rangle$ for $t \in \left[0, \frac{9\pi}{4}\right]$.

Problem 6 (10 points)

Consider the vector field in space, $\vec{F} = \langle x, y, z \rangle$, and the curve parametrized by $\vec{r}(t) = \langle \cos(t), \sin(t), \cos(t) \rangle$, fot $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is shown in the figure. Let C be the portion of the curve for $t \in \left[-\frac{\pi}{4}, \frac{\pi}{3}\right]$.

Find: $\displaystyle\int_C \vec{F} \cdot d\vec{r}$.



END OF EXAM 3