SARAH • LAWRENCE • COLLEGE

MATH 3614: COURSE SYLLABUS FALL 2016

Mathematics with Complex Variables: A Visual Approach

Level: Intermediate Updated: 9.4.2016

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Meeting Times

Seminars: Mondays & Wednesdays – 3:35 pm - 5:00 pm – Science Center – Room 201.
Individual student conferences to be scheduled on alternating weeks.

What is this class?

Once described as "that amphibian between existence and nonexistence" by Leibniz, the so-called "complex numbers," $z = x + iy$, where x and y are real numbers and $i = \sqrt{-1}$ is the "imaginary unit," were met with suspicion and hostility for almost two and a half centuries. These numbers have since proven to have a profound impact on the whole of mathematics. Once accepted, however, the development of a beautiful new theory of how to do calculus with such numbers was astonishingly rapid going from birth (in 1814) to maturity (in 1851) in less than 40 years! After this intense period of investigation, any lingering suspicions held by the scientific community over the reality of complex numbers were subsequently squashed due to the amazing utility of this "new calculus" in mathematics, physics, engineering, and elsewhere.

We'll learn the meaning of complex multiplication, exponentiation, as in Euler's famous equation $e^{i\pi} = -1$, and the associated geometry of these numbers. We'll study complex functions and their power series, and learn the many deep properties of Möbius transformations. We'll explore differentiation of complex functions and learn how to integrate them in the complex plane. We'll see the easiest proof of the Fundamental Theorem of Algebra, which says that every algebraic equation has a solution as long as you allow complex numbers. Numerous applications to other fields of mathematics (such as number theory or non-euclidean geometry), physics, and engineering can also be explored in seminar and in conference work according to the tastes and wishes of the students.

Learning Goals

To develop an understanding and appreciation of the mathematical concepts and tools in Complex Analysis (see outline below). Additionally, by the end of the semester students will be able to:

- use of appropriate mathematical terminology and notation.
- visualize and compute with complex numbers and functions.
- visualize, compute and interpretate complex derivatives.
- visualize, compute and interpretate complex integrals using Cauchy's theorem(s).
- manipulate complex infinite power and Laurent series.
- develop mathematical proof writing skills.

Who should take this class?

This seminar is a must for students interested in mathematics, physics, engineering, and any student satisfying the prerequisites wishing to broaden and enrich the life of the mind.

Prerequisites

Prerequisites: Successful completion of Calculus II or the equivalent (a score of 4 or 5 on the Calculus BC AP test) is required; completion of an intermediate-level mathematics course (e.g. Multivariable Calculus, Linear Algebra, or Discrete Mathematics) is recommended.

Required Texts

- (1) *Complex Analysis: A First Course with Applications*, Third Edition, by Zill and Shanahan.
- (2) *WebAssign* Access (for online HW).
- (3) *An Imaginary Tale: The story of* $\sqrt{-1}$, by Paul Nahin.
- (4) *Visual Complex Analysis*, by Tristan Needham (OPTIONAL!).

A few comments about the required texts.

- (1) This will be our main text and we'll cover almost all of it. Students are not required to purchase a physical copy of the book as an electronic version is provided at no additional cost with a purchase of access to WebAssign–Note bene: access to the ebook ends after the semester ends! However, I strongly recommend students buy a physical copy of the textbook.
- (2) WebAssign access can be purchased directly online (with a credit card) or with the textbook (if bought with the textbook, don't lose this "key code"!!!)
- (2) We will begin the course by reading the first three chapters from "An Imaginary Tale" in the first week of class so be sure to have a copy before the first day of class! A copy will be place on reserve at the library. After the first three chapters, you can read the other chapters for enjoyment in like a day or two after the semester is done or just keep reading it before jumping into the official textbook by Zill & Shanahan. Chapter 4 is really charming and hopefully will get you hooked on the subject. You can skip Ch 5, 6, & 7. If you want to preview the theory of Complex Analysis early, then read/skim chapter 7. Let anything tricky wash over you ;-)
- (3) I will provide selections from this textbook via MySLC. A copy will be place on reserve at the library so I recommend you skim through it at the library first. If you fall in love, by all means, buy it and maybe study from it in conference or after the basic course is done. This book has a lot of material. Enough for two semesters, and maybe more. It can be frustrating sometimes so be patient and find comfort in the fact that everyone probably struggles with this book early. Finally, I'll note that there are lots of potential topics for conference work!

Required Calculator

None.

Expectations

Course Readings

For each seminar meeting there will be an assigned reading from the course text or a supplemental handout. As this is a lecture-free course, these readings will form the basis of our seminar discussions. In preparation for each seminar meeting, students absolutely must complete the assigned reading and be fully prepared to discuss the ideas encountered in the readings and/or work out problems alone or in groups at

the blackboard. See "Course Readings and Seminar Exercises" hand-out (to be supplied on MySLC) for the scheduling of readings.

Reflection pieces

The ability to read mathematics successfully–for deep understanding and long term retention–is a skill that requires considerable effort to develop. It is also a skill that is often not developed in traditional high school courses. In this course you will have much opportunity to improve your technical reading/learning skills. As a first step, please consult the "Suggestions for Effective Reading of Mathematics" document (to be supplied on MySLC).

Students are required to prepare a summary (as little as a few sentences but sometimes they may be a paragraph or two; but no more!), a list of key terms and a question(s) (consult the "Suggestions for Effective Reading of Mathematics" document; to be supplied on MySLC) for each section of assigned reading in advance of each seminar meeting called "Reflection Pieces." These are due online two hours before seminar.

Online Problem Sets

In addition to completing the daily readings, students are required to prepare a short collection of exercises online using WebAssign, which may be worked on alone or in groups. Due dates will be provided through WebAssign. I will post the "class key" on MySLC for signing up for our section in WebAssign. Note: You may sign up for WebAssign for free for the first two weeks, after that you will be required to pay for the service with a credit card (or if you buy the book with WebAssign access bundled together then you will use the code provided).

Hand-in Homeworks

In addition to completing the daily readings and the online problem sets, there will be weekly problem sets to be written up and done "by hand" called Hand-In Homeworks (HHWS) and/or Take-Home Examinations (THEs).

HHWs will be due every Saturday at 12:01 p.m. (Noon) Problem sets are to be submitted electronically (scanned or typed up). Each student will have a designated electronic file folder for submitting coursework. You can expect to receive a link to the shared folder via an email invitation during the first week of class. I encourage you to work collaboratively, so long as you write your problem set yourself in your own words (fingers on keys). It's a good idea to start early, and be sure to attempt a problem on your own before discussing it with others. Any assistance or source material that helped you complete a problem set must be properly acknowledged at the top of your assignment.

Mathematical Typing with LaTeX

A least one HHW assignment must be created using the software LaTeX. It will ask that you submit both the ".tex" file and the corresponding ".pdf" output. More details will be given during the second or third week of classes.

There are no in-class quizzes or exams in this course. Reflection pieces, WebAssign problems, and Hand-in Homeworks permit assessment of student progress in learning the topics in this course.

Conference Work

Each student in the course will be expected to design and complete an independent project for conference work. Individual conferences will be held biweekly. Student conference work may be dedicated to a deeper

investigation of a single concept, topic, or application of Complex Analysis covered in seminar, the textbooks, or elsewhere. Keep in mind the following criteria: Mathematical Content, Effort, Comprehensibility, Degree of Difficulty, and Style and Presentation.

The idea is for you to find some aspect of Complex Analysis that is interesting enough to you that you want to explore further and spend significant time understanding thoroughly and presenting to me. Obviously, many topics in Complex Analysis have interesting visual descriptions and geometric interpretations. I also want the project to reflect the individual student's interest so I am trying to be as flexible as possible in what your project/product can be. I strongly encourage the use of LaTeXfor writing up the final work (if applicable).

Important dates[1]:
10/7 Friday: Conference proposal
11/16 Wednesday: Conference draft
12/16 Friday: Conference project and presentation

Portfolio

At the end of the semester, students will turn-in a "hard copy" and an "e-copy" of a class portfolio containing the contents of their work throughout the term in a polished, corrected, and updated form. The portfolio will include all materials from Reflection Pieces, Hand-in Homework, Conference Work plus a few other small bound in a three-ring binder with a table of contents. More details will be given in seminar or via MySLC later in the term.

Late Work Policy

You may ask for extensions on WebAssign homework directly from WA. I reserve the right to deny extensions for any reason. Late work will not be accepted for other assignments. However, two extensions will be granted to any student who requests an extension at least 24 hours in advance of the original deadline. Extension requests can be presented in person, over the phone, or via email. Except in unusual circumstances, each student will be granted only two deadline extension during the semester.

Additional Help

I encourage students who are having difficulty with the course material to meet with me for individualized help. Students are also encouraged to develop and maintain an email dialogue with me so that I may provide more timely assistance with smaller-scale questions.

Evaluations

At the end of the semester an individual course evaluation and course grade will be given to each student. This evaluation will be based primarily on the studnets level of preparation for seminar, contributions to seminar discussion and quality of work on the problem sets, and conference project. There will be no formal examinations in this course.

Self-Evaluations

Introspection is an important component of the learning process: a student's evaluation of his or her own work is as important, if not more important, than any teacher's evaluation. As such, students will be required to write a brief statement of self-evaluation on each assignment (problem sets and conference project) and an additional, more comprehensive self-evaluation at the end of the term.

[1] Dates subject to change

Attendance

Both lecture and discussion session attendance is absolutely mandatory. Students who miss more than two classes or discussion sessions (without a documented reason) run the risk of receiving reduced course grade. Number of classes missed and number of classes with significant tardiness will be indicated on the course evaluation. If a class is missed, the student is responsible for obtaining class notes and assignments, and the student is expected to be fully prepared for the next class session.

Note: Except in cases of emergency or a full 24 hour advanced notice, there will be no rescheduling of missed conferences. However, when unavoidable situations occur, students may request an alternative conference time in advance of the regularly scheduled conference time.

Science & Math. Third Programs

Qualified students may enroll in this course as part of a Science and Mathematics third program. This registration option allows students to enroll in two science and mathematics courses that together constitute a third of the enrolling student's registration for the term. This seminar can be combined under this third program with another seminar or a lecture course and either a yearlong or semester course. Interested students should consult with the professors of both courses, and permission of both professors is required to enroll in the program.

Disability Services

If you have a disability that may interfere with your ability to participate in the activities, coursework, or assessment of the objectives of this course, you may be entitled to accommodations. Please contact Polly Waldman, Associate Dean of Studies and Disabilities Services, in Westlands 207C or x2235 or pwaldman@sarahlawrence.edu. Under the Americans with Disabilities Act and Section 504 of the Vocational Rehabilitation Act of 1973, all students, with or without disabilities, are entitled to equal access to the programs and activities of Sarah Lawrence College.

Course Outline

This is broad course outline of the topics covered. I will post a separate document with the entire course schedule of readings on MySLC shortly. Consult MySLC for updates.

• Pre-Course

☐ Watch "A Whirwind Tour of Complex Analysis" (5 short videos) on YouTube:
https://www.youtube.com/watch?v=b0DteY-SzcQ.
Watch before semester starts, then re-watch & re-watch & re-watch :-)

• Historical Introduction

☐ Chapter 1, 2, & 3 of "An Imaginary Tale" in the first week.

• Zill & Shanahan textbook

☐ Chapters 1: Complex Numbers and the Complex Plane
☐ Chapters 2: Complex Functions and Mappings
☐ Chapters 3: Analytic Functions
☐ Chapters 4: Elementary Functions

⋆ Midterm on Chapters 1-4? (Negotiable!)

☐ Chapters 5: Integration in the Complex Plane
☐ Chapters 6: Series and Residues
☐ Chapters 7: Conformal Mappings

⋆ Final (take-home or oral)? (Negotiable!)

• Additional readings TBA

⋆ Postscript: A Brave new World with Complex Numbers! Conference presentations.