Calculus II - Spring 2019

Instructor: *Dr. Basilio*

Lab 1

Differential Equations, Part 1

Slope Fields and Solving Differential Equations

Due: Tuessday, May 28 by 11:59pm via CoCalc

Objectives

- Learn the syntax for other functions needed for calculus
- Learn how to plot simple slope fields using `SageMath`
- Learn how to solve simple differential equations using `SageMath`
- Learn to plot the particular solutions together with a slope field using `SageMath`

Section 0. Create your lab document.

In your Lab_1 folder, create a new `SAGE worksheet` (.sagewa) file titled:

```
"Math5B-Lab_1-LASTNAME_FIRSTNAME-S19.sagews"
```

At the beginning of your document, type the following comments:

```
# Math 5B - Calc II - Spring 2019
# Lab_1
# Due: Tuesday, May 28 @11:59pm via CoCalc

# ()Your Last Name, Your First Name)

# Assignment

# Problem 1
```

Be sure to use comments to label each problem and it's parts.

Section 1. More functions

Problem 1: Look up the syntax for the following functions:

If not asked to graph, then just type the syntax for the function.

a. $|x|$

b. $\log_{10}(x)$

c. $\tan(x)$, $\sec(x)$, $\csc(x)$, $\cot(x)$

d. $\sin^{-1}(x) = \arcsin(x)$, $\tan^{-1}(x) = \arctan(x)$

e. Graph $y = \arctan(x)$ and evaluate $\lim_{x \to \infty} \arctan(x)$

f. $\sinh(x)$, $\cosh(x)$, $\tanh(x)$

g. Graph $y = \tanh(x)$ and find an appropriate scale for the axes that shows all important features of the graph.

Section 2. Slope Fields in Sage

Recall that solving differential equations (DEs) is very hard and it can be difficult to solve one with an exact formula.
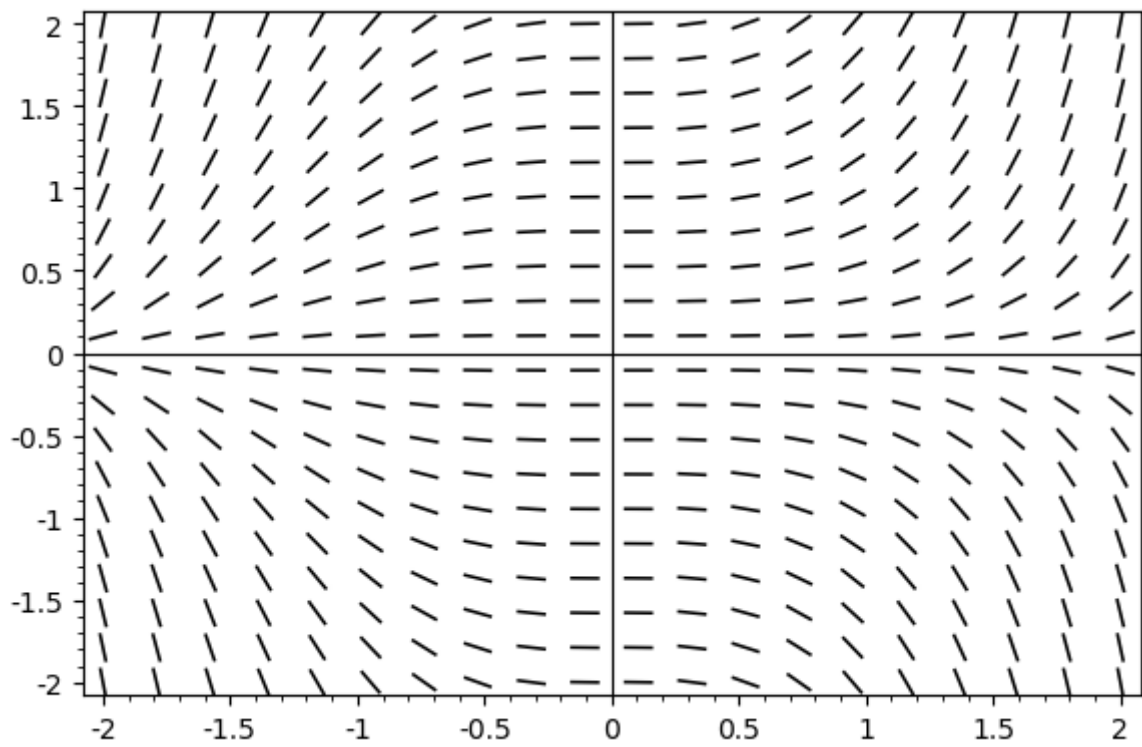
Slope fields (or direction fields), are visual plots of the DE: $\frac{dy}{dx} = F(x, y)$.

These are difficult to graph by hand but a computer can do it easily.
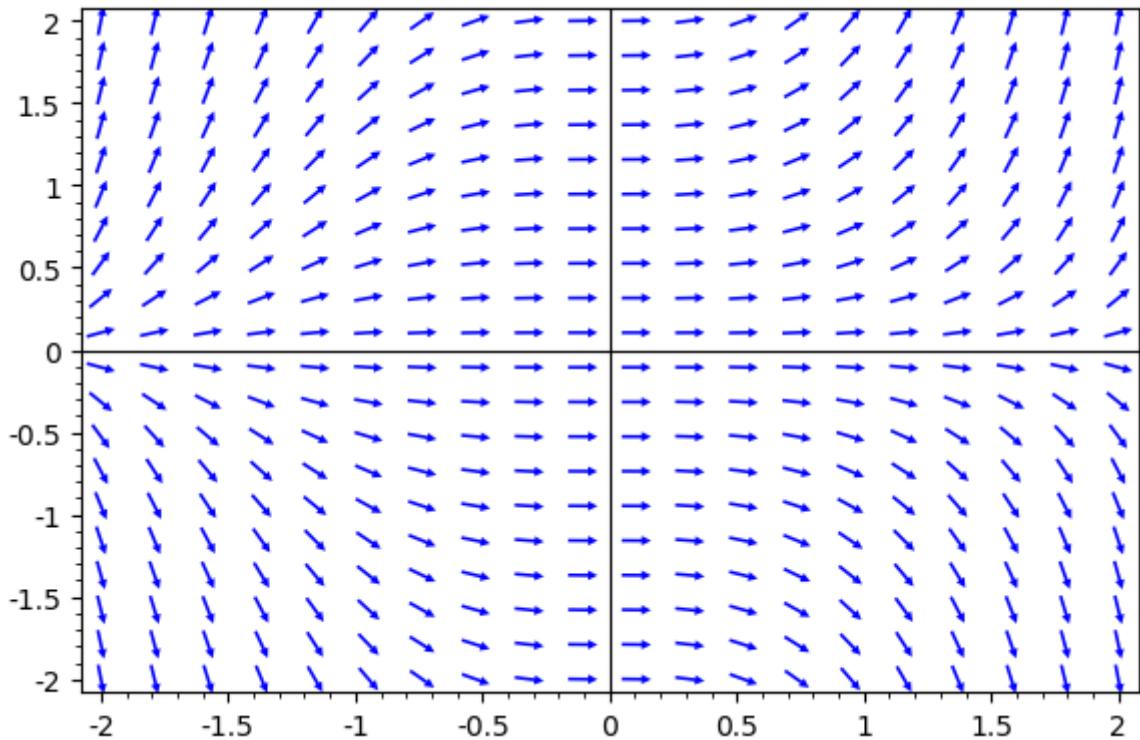
Here's how:

```
# Slope fields
x,y = var('x y') # define variables x and y
F(x,y) = x^2*y # define the function F(x,y)
plot_slope_field( F(x,y), (x,-2,2), (y,-2,2)) # Note: you can change the
size of the window.
```

```
# Customize the slope field:
plot_slope_field( F(x,y), (x,-2,2), (y,-2,2), color='blue', headaxisleng
th=3, headlength=3)
```

Problem 2. Plot the following slope fields:

You will need to modify the x- and y-values in the graphing window so that all interesting features are shown.

a. $\dfrac{dy}{dx} = \sin(x) \cdot \sin(y)$

b. $\dfrac{dy}{dx} = x + y - 1$

c. $y' = x(2 - y)$

d. $y' = 2 - y$

e. $y' = x^2 y - \dfrac{1}{2}y^2$

f. $\dfrac{dy}{dx} = \cos(x + y)$

Section 3. Solving Differential Equations

We now show how to solve differential equations using `SAGE`

We'll start with solving: $y' = y$.

If you recall from Chapter 6 and 9, this is the law of natural growth with constant $k = 1$.

We know the general solution is $y(x) = Ce^x$ (again, with $k = 1$).

Let's see how to solve it in `SAGE` .

```
In [114]:  # Solving a DE
           x = var('x') # define the independent variable
           y = function('y')(x) # define y as a function of x
           DE = diff(y,x) - y # define the differerntial equation: y'-y=0.
               # Note: We want to put everything on one side except the "=0"
           gen_sol = desolve(DE, y) # solve the differential equation DE and store
            it to "gen_sol"
               # the ",y" asks SAGE to solve for y.
           show(gen_sol)
```

Out[114]: Ce^x

```
In [115]:  # Solving a DE with initial conditions
           x = var('x') # define the independent variable
           y = function('y')(x) # define y as a function of x
           DE = diff(y,x) - y # define the differerntial equation: y'-y=0. We will
            want to put everything on one side but not the "=0"

           part_sol = desolve(DE, y, ics=[0,1]) # solve the differential equation D
           E with initial conditions (the "ics")
               # ics=[0,1] means x=0 and y=1
           show(part_sol)

           part_sol_2 = desolve(DE, y, ics=[0,1.5]) # solve the differential equati
           on DE with initial conditions (the "ics")
               # ics=[0,1] means x=0 and y=1
           show(part_sol_2)
```

Out[115]: e^x

Out[115]: $\dfrac{3}{2}e^x$

In [116]:
```
# Plotting slope field and particular solution together
x,y = var('x y')
# slope field for: y'=y so F(x,y)=y
SF = plot_slope_field( y, (x,-2,2), (y,-2,2), color='blue', headaxisleng
th=3, headlength=3) # SF = slope field

PS1 = plot(part_sol, (x,-2,2), color='red', thickness=2.5) # PS = partic
ular solution with C=1

PS2 = plot(part_sol_2, (x,-2,2), color='red', thickness=2.5) # PS2 = par
ticular solution 2 with C=1.5

#note: by defining SF, PS1 and PS2 they are stored in SAGE. To plot them
we will use 'show()' command
show(SF+PS1+PS2)
```
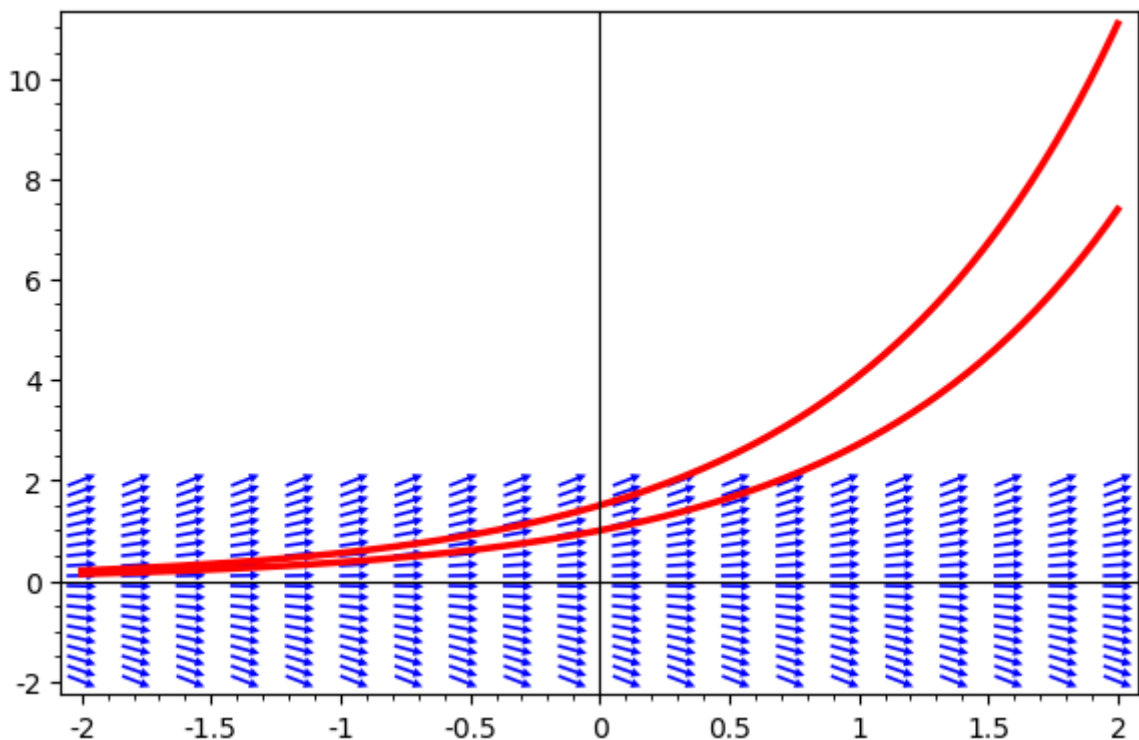
Out[116]:

Notice the graph is correct but doesn't look good. We can manipulate the windown so that they all match.

We'll keep the slope field window from $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$, but change the window for the particular solutions so that the red curves don't go past $y = 2$.

In [118]:
```
# Plotting slope field and general solution together (with better window
and with point emphasized)
x,y = var('x y')
# slope field for: y'=y so F(x,y)=y
SF = plot_slope_field( y, (x,-2,2), (y,-2,2), color='blue', headaxisleng
th=3, headlength=3) # SF = slope field

PS = plot(part_sol, (x,-2,2), ymax=2, color='red', thickness=2.5) # PS =
particular solution with C=1

PS2 = plot(part_sol_2, (x,-2,2), ymax=2, color='red', thickness=2.5) # P
S = particular solution with C=1.5

# Let's add the points too:
Pt = point((0,1), size=50, color='red') # add the point with IC
Pt2 = point((0,1.5), size=50, color='red') # add point with IC2

#note: by defining SF and PS etc they are stored in SAGE. To plot them w
e will use 'show()' command
show(SF+PS+PS2+Pt+Pt2)
```
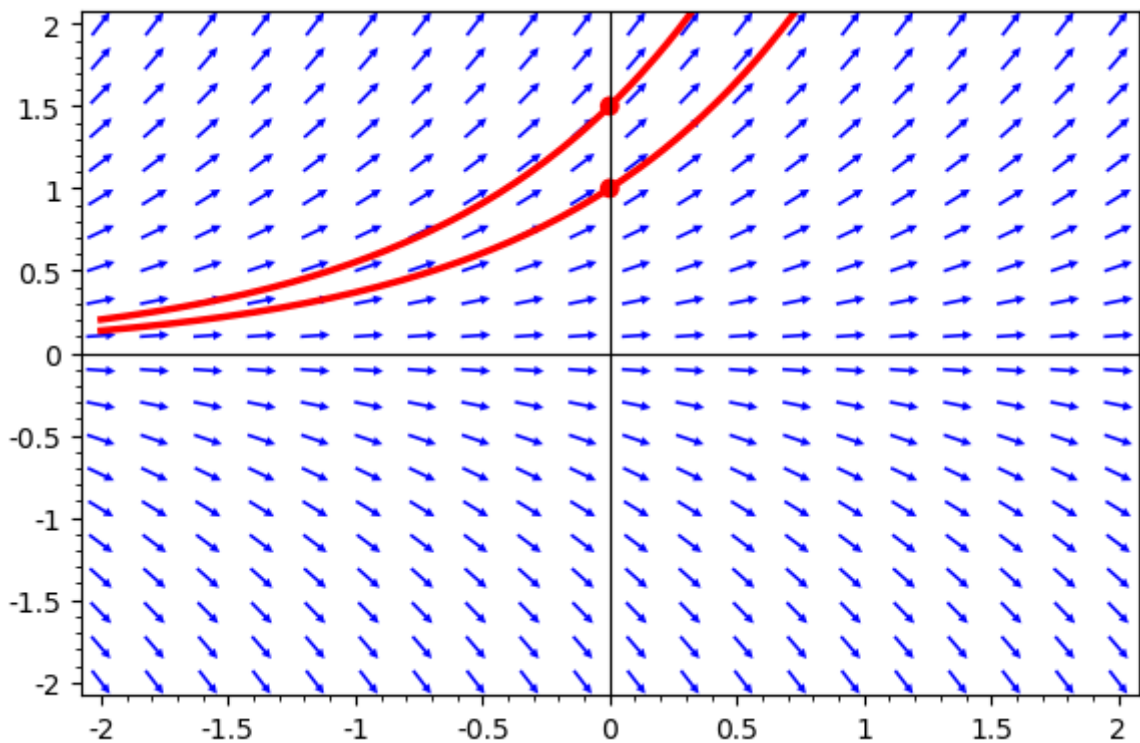
Out[118]:

Problem 3. Solving Differential Equations and Graphing a Particular solution

Consider (DE): $\dfrac{dy}{dx} = -y - \sin(x)$.

a. Plot the slope field for the DE with window: $-3 \le x \le 3$ and $-3 \le y \le 3$.

b. Find the general solution to the differential equation.

c. Find the particular solutions passing through:

i. $(-2, -2)$ ii. $(-1, -2)$ iii. $(0, -2)$ iv. $(1, -2)$ v. $(2, -2)$ vi. $(-2, 2)$

d. Plot all five particular solutions found in part (c) in one window. Include the points of the initial conditions.

Bonus:

Extra-Credit

Plot the slope field of: $\dfrac{dy}{dx} = x - y + 1$ with window $-3 \le x \le 3$ and $-3 \le y \le 3$.

Include in your plot the 12 particular solutions passing through:

$(-2, -2), (-1, -2), (0, -2), (1, -2), (2, -2)$

and

$(-2.8, 0), (-2.8, 1), (-2.8, 2)$

and

$(-2, 2), (-1, 2), (0, 2), (1, 2)$.

Include in your plot the 12 points as well.