Review of Theory: Calc I & II

A few Important Definitions

Give precise definitions to the following.

1. Limit of f as x approaches a $\lim_{x\to a} f(x) = L$
2. Continuity of f at $x = a$
3. Limit of f as x approaches $+\infty$: $\lim_{x\to+\infty} f(x) = L$
4. Derivative of f at $x = a$: $f'(a) = \frac{df}{dx}\Big|_{x=a} = \frac{dy}{dx} = \dot{s}$
5. Equation of the tangent line to f at the point $P = (a, f(a))$.
6. Anti-derivative (or, indefinite integral) of f, $F(x) = \int f(x)dx$.
7. Definite integral of f, $\int_a^b f(x)dx$.

   This one is complicated: to help, here's an outline:

- Step 1: Partition of $[a, b]$ with n subintervals $[x_{i-1}, x_i]$, set Δx_i.
- Step 2: Pick $c_i \in [x_{i-1}, x_i]$.
- Step 3: Riemann sum: $\sum_{i=1}^{n} f(c_i)\Delta x_i$.
- Step 4: Definite Integral is the limit of Riemann sums, write this using math notatation.

8. Explain the difference between a definite integral and an anti-derivative

A few Important Theorems

Give a precise or "slogan form" of the following theorems.

1. Special Limits:
   (a) $\lim_{x\to\pm\infty} \frac{1}{x} =$ (b) $\lim_{x\to 0} \frac{\sin(x)}{x} =$ (c) $\lim_{x\to 0} \frac{1-\cos(x)}{x} =$

2. Derivative Rules:
   (a) Power Rule (b) Product Rule (c) Quotient Rule (d) Chain Rule Rule
   (e) $\frac{d}{dx}\big[\sin(x)\big] =$ (f) $\frac{d}{dx}\big[\cos(x)\big] =$ (g) $\frac{d}{dx}\big[\tan(x)\big] =$
3. Derivative Tests (DTs)
   (a) ID Test (Increasing/Decreasing Test (b) First DT for Local Extrema
   (c) Second DT for Concavity (d) Second DT for Local Extrema
4. ****Fundamental Theorems of Calculus****
   (a) Part I (b) Part II
5. (Optinal) Serious bonus points if you remember these without looking them up!!
   (a) Intermediate Value Theorem (b) Extreme Value Theorem (c) Mean Value Theorem

A few Important Techniques

Know the following techniques to solve problems.

1. Calculate limits using limit rules and theorems
2. Compute tangent lines and use these to approximate a function
3. Implicit Differentiation 4. Related Rates
5. Calculate complicated derivatives using Derivative Rules (DRs)
6. Sketching a curve using f' and f''
7. ***Optimization Problems*** The largest…/The smallest…
8. Calculate anti-derivatives
9. Calculate definite integrals
10. Calculate integrals using *u-substittution*
11. Calculate areas using definite integrals
12. (Optinal) Calculate Riemann sums to approximate a definite integral using
    (a) left-endpoints (b) right-endpoints (c) midpoints (d) trapezoids
13. Integration Techniques: Integration by Parts, Trig Substitution, (Partial Fractions)
14. Integate: $\cos^2(x)$, $\sin^2(x)$

Practice Problems

Practice Problems from Calc I

Solve the following.

1. Some algebra
   (a) Long division: divide $x^4 + 2x^2 - 4x + 6$ by $x^2 + x + 3$

   (b) Complete the square: $x^2 + 5x - 3$

2. Some trig
   (a) Fill-out a complete unit circle

   (b) Find all values of x where $\cos(x) = \dfrac{\sqrt{3}}{2}$

3. If $f(x) = \dfrac{\cos\left((2x^2 - 3)^{94}\right)}{7x^2 \cdot \tan(6x)} + \pi^2$, compute $f'(x)$

4. Find the equation of the tangent line of $g(x) = x\sin(x)$ at the point where $x = \pi/3$.

5. Compute: $\displaystyle\int \left(\pi\cos(x) + (x^3 - x^5)^2\right) dx$

6. Compute: $\displaystyle\int \left(3(x^2+1)\sec^2(x^3+3x)\right) dx$

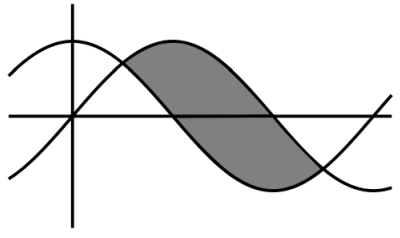
7. Evaluate: $\displaystyle\int_{-1}^{1} x^{99}\, dx$

8. Find: $\dfrac{d}{dx}\displaystyle\int_0^x \tan^3(t^2)\, dt$

9. Find $\frac{dy}{dx}$ as a function of x and y given that

$$xy + y^2 + x^2 = 3$$

10. Find the area above $y = \cos(x)$ and below $y = \sin(x)$ between their first two intersections (see diagram).



11. Given the cost and revenue functions $C(x) = 2x+10$ and $R(x) = -2x^2+20x$ that represent the number of dollars spent or made, respectively, on the sale of x units of a certain commodity. What production levels maximize profits? Recall that the profit function is $P(x) = R(x) - C(x)$.

12. A printer need to make a poster that will have a total area of 200 in^2 and will have 1 inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom (draw a picture!). What dimensions will give the largest printed area?

Practice Problems from Calc II

Solve the following.

1. Some graphs
   (a) Graph: $f(x) = e^x$ and $g(x) = e^{-x}$
   (b) Graph: $f(x) = \ln(x)$
   (c) Graph: $f(x) = \sin^{-1}(x)$ and $g(x) = \tan^{-1}(x)$
2. Some limits
   (a) Find: $\lim\limits_{x\to 0} \dfrac{e^x}{x^2}$
   (b) Find: $\lim\limits_{x\to 0^+} x\ln(x)$
3. Some derivatives
   (a) Find: $\dfrac{d}{dx}\left[\dfrac{e^{2x}\sinh(x)}{\ln(1-x)}\right]$
   (b) Find: $\dfrac{d}{dx}\left[\arctan(x)\right]$
   (c) Find: $\dfrac{d}{dx}\left[\arcsin(x)\right]$
4. Some integrals
   (a) Find: $\displaystyle\int \cos^2(x)\,dx$
   (b) Find: $\displaystyle\int \sin^2(x)\,dx$
   (c) Find: $\displaystyle\int xe^x\,dx$
   (d) Find: $\displaystyle\int \sqrt{1-x^2}\,dx$