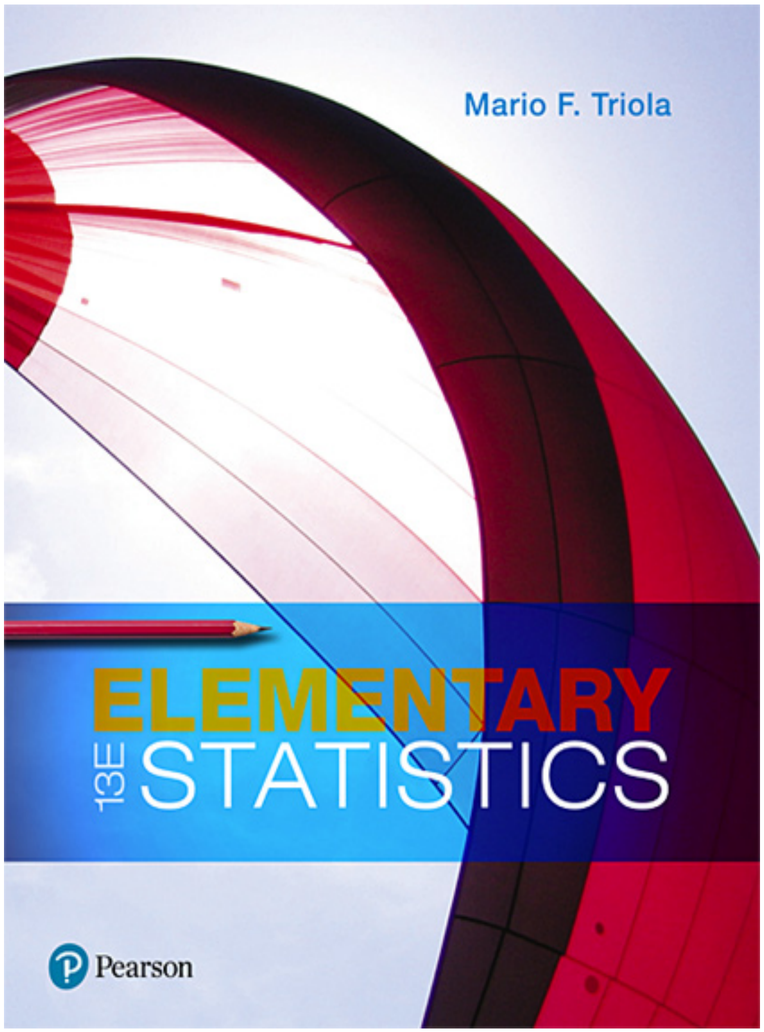
Elementary Statistics

Thirteenth Edition



Chapter 4

Probability

Probability

4-1 Basic Concepts of Probability

4-2 Addition Rule and Multiplication Rule

4-3 Complements and Conditional Probability, and Bayes' Theorem

4-4 Counting

4-5 Probabilities Through Simulations (available at TriloaStats.com)

Key Concept

The single most important objective of this section is to learn how to **interpret** probability values, which are expressed as values between 0 and 1. A small probability, such as 0.001, corresponds to an event that rarely occurs.

Next are **odds** and how they relate to probabilities. Odds are commonly used in situations such as lotteries and gambling.

Basics of Probability

- An **event** is any collection of results or outcomes of a procedure.
- A **simple event** is an outcome or an event that cannot be further broken down into simpler components.
- The **sample space** for a procedure consists of all possible **simple** events. That is, the sample space consists of all outcomes that cannot be broken down any further.

Example: Simple Events and Sample Spaces (1 of 5)

In the following display, we use “b” to denote a baby boy and “g” to denote a baby girl.

Procedure	Example of Event	Sample Space: Complete List of Simple Events
Single birth	1 girl (simple event)	{b, g}
3 births	2 boys and 1 girl (bbg, bgb, and gbb are all simple events resulting in 2 boys and 1 girl)	{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg}

Example: Simple Events and Sample Spaces (2 of 5)

Solution

Simple Events:

- With one birth, the result of 1 girl is a **simple event** and the result of 1 boy is another simple event. They are individual simple events because they cannot be broken down any further.

Example: Simple Events and Sample Spaces (3 of 5)

Solution

Simple Events:

- With three births, the result of 2 girls followed by a boy (ggb) is a simple event.
- When rolling a single die, the outcome of 5 is a simple event, but the outcome of an even number is not a simple event.

Example: Simple Events and Sample Spaces (4 of 5)

Solution

Not a Simple Event: With three births, the event of “2 girls and 1 boy” is **not a simple event** because it can occur with these different simple events: ggb, gbg, bgg.

Example: Simple Events and Sample Spaces (5 of 5)

Solution

Sample Space: With three births, the **sample space** consists of the eight different simple events listed in the above table.

Procedure	Example of Event	Sample Space: Complete List of Simple Events
Single birth	1 girl (simple event)	{b, g}
3 births	2 boys and 1 girl (bbg, bgb, and gbb are all simple events resulting in 2 boys and 1 girl)	{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg}

Three Common Approaches to Finding the Probability of an Event (1 of 6)

Notation for Probabilities

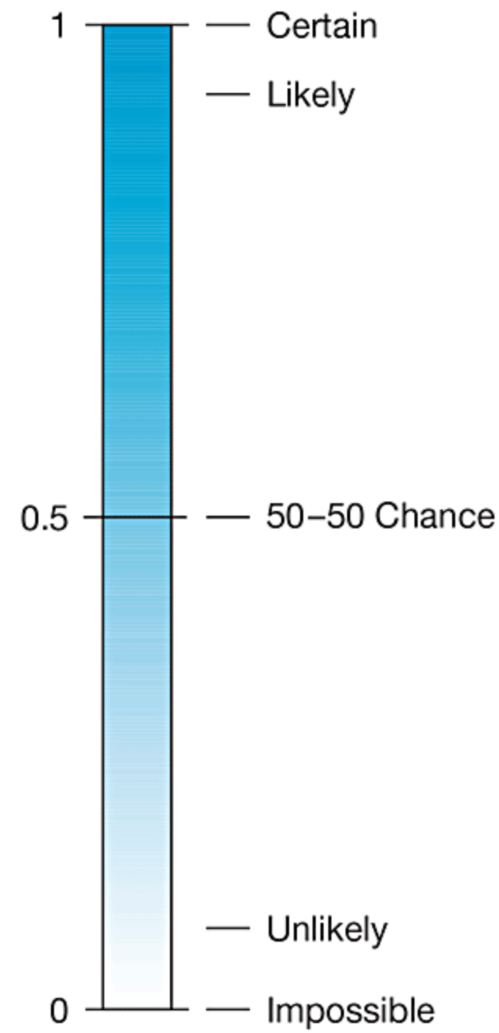
P denotes a probability.

A, B, and C denote specific events.

$P(A)$ denotes the "probability of event A occurring."

Three Common Approaches to Finding the Probability of an Event (2 of 6)

Possible values of probabilities and the more familiar and common expressions of likelihood



Three Common Approaches to Finding the Probability of an Event (3 of 6)

The following three approaches for finding probabilities result in values between 0 and 1: $0 \leq P(A) \leq 1$.

Three Common Approaches to Finding the Probability of an Event (4 of 6)

1. **Relative Frequency Approximation of Probability**
   Conduct (or observe) a procedure and count the number of times that event *A* occurs. *P*(*A*) is then **approximated** as follows:

$$P(A) = \frac{\text{number of times } A \text{ occurred}}{\text{number of times the procedure repeated}}$$

Three Common Approaches to Finding the Probability of an Event (5 of 6)

2. **Classical Approach to Probability (Requires Equally Likely Outcomes)** If a procedure has *n* different simple events that are **equally likely**, and if event *A* can occur in *s* different ways, then

$$P(A) = \frac{\text{number of ways } A \text{ occurs}}{\text{number of different simple events}} = \frac{s}{n}$$

Caution When using the classical approach, always confirm that the outcomes are **equally likely**.

Three Common Approaches to Finding the Probability of an Event (6 of 6)

3. **Subjective Probabilities** $P(A)$, the probability of event A, is **estimated** by using knowledge of the relevant circumstances.

Simulations

- **Simulations**
  - Sometimes none of the preceding three approaches can be used. A **simulation** of a procedure is a process that behaves in the same ways as the procedure itself so that similar results are produced. Probabilities can sometimes be found by using a simulation.

Rounding Probabilities

When expressing the value of a probability, either give the **exact** fraction or decimal or round off final decimal results to **three** significant digits.

(**Suggestion:** When a probability is not a simple fraction such as $\frac{2}{3}$ or $\frac{5}{9}$, express it as a decimal so that the number can be better understood.)

Law of Large Numbers (1 of 2)

- **Law of Large Numbers**
  - As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

Law of Large Numbers (2 of 2)

- **Law of Large Numbers**

CAUTIONS

1. The law of large numbers applies to behavior over a large number of trials, and it does not apply to any one individual outcome.
2. If we know nothing about the likelihood of different possible outcomes, we should not assume that they are equally likely. The actual probability depends on factors such as the amount of preparation and the difficulty of the test.

Example: Relative Frequency: Skydiving (1 of 3)

Find the probability of dying when making a skydiving jump.

Example: Relative Frequency: Skydiving (2 of 3)

Solution

In a recent year, there were about 3,000,000 skydiving jumps and 21 of them resulted in deaths. We use the relative frequency approach as follows:

$$P(\text{skydiving death}) = \frac{\text{number of skydiving deaths}}{\text{number of skydiving jumps}}$$

$$= \frac{21}{3{,}000{,}0000} = 0.000007$$

Example: Relative Frequency: Skydiving (3 of 3)

Solution

Here the classical approach cannot be used because the two outcomes (dying, surviving) are not equally likely.

A subjective probability can be estimated in the absence of historical data.

Example: Texting and Driving (1 of 4)

In a study of U.S. high school drivers, it was found that 3785 texted while driving during the previous 30 days, and 4720 did not text while driving during that same time period (based on data from "Texting While Driving . . . ," by Olsen, Shults, Eaton, **Pediatrics**, Vol. 131, No. 6). Based on these results, if a high school driver is randomly selected, find the probability that he or she texted while driving during the previous 30 days.

Example: Texting and Driving (2 of 4)

Solution

Instead of trying to determine an answer directly from the given statement, first summarize the information in a format that allows clear understanding, such as this format:

3785	texted while driving
4720	did not text while driving
8505	total number of drivers in the sample

Example: Texting and Driving (3 of 4)

Solution

We can now use the relative frequency approach as follows:

$$P(\text{texting while driving}) = \frac{\text{drivers who texted while driving}}{\text{number of drivers in the sample}} = \frac{3785}{8505} = 0.445$$

Example: Texting and Driving (4 of 4)

Interpretation

There is a 0.445 probability that if a high school driver is randomly selected, he or she texted while driving during the previous 30 days.

Complementary Events

- **Complement**
  - The **complement** of event *A*, denoted by $\bar{A}$, consists of all outcomes in which event *A* does **not** occur.

Example: Complement of Death from Skydiving (1 of 2)

In a recent year, there were 3,000,000 skydiving jumps and 21 of them resulted in death. Find the probability of **not** dying when making a skydiving jump.

Example: Complement of Death from Skydiving (2 of 2)

Solution

Among 3,000,000 jumps there were 21 deaths, so it follows that the other 2,999,979 jumps were survived. We get

$$P(\text{not dying when skydiving}) = \frac{2{,}999{,}979}{3{,}000{,}000} = 0.999993$$

The probability of **not** dying when making a skydiving jump is 0.999993.

Identifying Significant Results with Probabilities (1 of 3)

The Rare Event Rule for Inferential Statistics

If, under a given assumption, the probability of a particular observed event is very small and the observed event occurs **significantly less than or significantly greater than** what we typically expect with that assumption, we conclude that the assumption is probably not correct.

Identifying Significant Results with Probabilities (2 of 3)

Using Probabilities to Determine When Results Are Significantly High or Significantly Low

- **Significantly high number of successes:** x successes among n trials is a **significantly high** number of successes if the probability of x or more successes is unlikely with a probability of 0.05 or less. That is, x is a significantly high number of successes if $P(x \text{ or more}) \leq 0.05$*.

*The value 0.05 is not absolutely rigid.

Identifying Significant Results with Probabilities (3 of 3)

Using Probabilities to Determine When Results Are Significantly High or Significantly Low

- **Significantly low number of successes:** x successes among n trials is a **significantly low** number of successes if the probability of x or fewer successes is unlikely with a probability of 0.05 or less. That is, x is a significantly low number of successes if $P(x \text{ or fewer}) \leq 0.05$*.

*The value 0.05 is not absolutely rigid.

Probability Review

- The probability of an event is a fraction or decimal number between 0 and 1 inclusive.
- The probability of an impossible event is 0.
- The probability of an event that is certain to occur is 1.
- Notation: $P(A)$ = the probability of event A.
- Notation: $P(\bar{A})$ = the probability that event A does **not** occur.

Odds (1 of 3)

- **Actual Odds Against**
  - The **actual odds against** event *A* occurring are the ratio $\frac{P(\bar{A})}{P(A)}$, usually expressed in the form of *a*:*b* (or "*a* to *b*"), where *a* and *b* are integers. (Reduce using the largest common factor; if *a* = 16 and *b* = 4, express the odds as 4:1 instead of 16:4.)

Odds (2 of 3)

- **Actual Odds in Favor**
  - The **actual odds in favor** of event *A* occurring are the ratio $\frac{P(A)}{P(\bar{A})}$ which is the reciprocal of the actual odds against that event. If the odds against an event are *a*:*b*, then the odds in favor are *b*:*a*.

Odds (3 of 3)

- **Payoff Odds**
  - The **payoff odds** against event *A* occurring are the ratio of net profit (if you win) to the amount bet:

    Payoff odds against event *A* = (net profit):(amount bet)

Example: Actual Odds Versus Payoff Odds (1 of 4)

If you bet $5 on the number 13 in roulette, your probability of winning is $\frac{1}{38}$, but the payoff odds are given by the casino as 35:1.

a. Find the actual odds against the outcome of 13.

b. How much net profit would you make if you win by betting $5 on 13?

c. If the casino was not operating for profit and the payoff odds were changed to match the actual odds against 13, how much would you win if the outcome were 13?

Example: Actual Odds Versus Payoff Odds (2 of 4)

Solution

a. With $P(13) = \frac{1}{38}$ and $P(\text{not } 13) = \frac{37}{38}$, , we get

$$\text{Actual odds against } 13 = \frac{P(\text{not } 13)}{P(13)}$$

$$= \frac{37/38}{1/38}$$

$$= \frac{37}{1} \text{ or } 37{:}1$$

Example: Actual Odds Versus Payoff Odds (3 of 4)

Solution

b. Because the casino payoff odds against 13 are 35:1, we have

$$35:1 = \text{(net profit)}:\text{(amount bet)}$$

So there is a \$35 profit for each \$1 bet. For a \$5 bet, the net profit is \$175 (which is 5 * \$35). The winning bettor would collect \$175 plus the original \$5 bet. After winning, the total amount collected would be \$180, for a net profit of \$175.

Example: Actual Odds Versus Payoff Odds (4 of 4)

Solution

c. If the casino were not operating for profit, the payoff odds would be changed to 37:1, which are the actual odds against the outcome of 13. With payoff odds of 37:1, there is a net profit of $37 for each $1 bet. For a $5 bet, the net profit would be $185. (The casino makes its profit by providing a profit of only $175 instead of the $185 that would be paid with a roulette game that is fair instead of favoring the casino.)