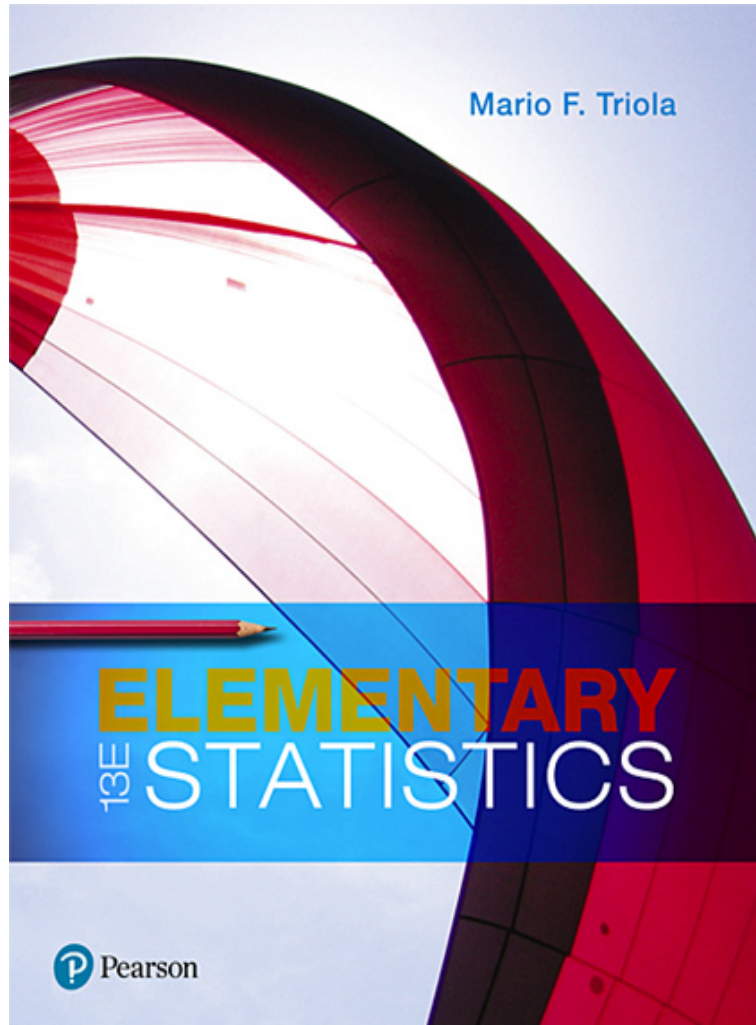


Elementary Statistics

Thirteenth Edition



Chapter 4 Probability

Probability

4-1 Basic Concepts of Probability

4-2 Addition Rule and Multiplication Rule

4-3 Complements and Conditional Probability, and Bayes' Theorem

4-4 Counting

4-5 Probabilities Through Simulations (available at TriloaStats.com)

Key Concept (1 of 2)

In this section we present the **addition rule** as a tool for finding $P(A \text{ or } B)$, which is the probability that either event A occurs or event B occurs (or they both occur) as the single outcome of a procedure. The word “or” in the addition rule is associated with the addition of probabilities.

Key Concept (2 of 2)

This section also presents the basic **multiplication rule** used for finding $P(A \text{ and } B)$, which is the probability that event A occurs and event B occurs. The word “and” in the multiplication rule is associated with the multiplication of probabilities.

Compound Event

- Compound Event
 - A **compound event** is any event combining two or more simple events.

Addition Rule

Notation for Addition Rule

$P(A \text{ or } B) = P(\text{in a single trial, event } A \text{ occurs or event } B \text{ occurs or they both occur})$

Intuitive Addition Rule

To find $P(A \text{ or } B)$, add the number of ways event A can occur and the number of ways event B can occur, but **add in such a way that every outcome is counted only once.** $P(A \text{ or } B)$ is equal to that sum, divided by the total number of outcomes in the sample space.

Formal Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

where $P(A \text{ and } B)$ denotes the probability that A and B both occur at the same time as an outcome in a trial of a procedure.

Disjoint Events and the Addition Rule

- Disjoint (or mutually exclusive)
 - Events A and B are **disjoint** (or **mutually exclusive**) if they cannot occur at the same time. (That is, disjoint events do not overlap.)

Example: Disjoint Events (1 of 2)

Disjoint events:

Event A—Randomly selecting someone for a clinical trial who is a male

Event B—Randomly selecting someone for a clinical trial who is a female (The selected person **cannot** be both.)

Example: Disjoint Events (2 of 2)

Events that are **not** disjoint:

Event A—Randomly selecting someone taking a statistics course

Event B—Randomly selecting someone who is a female (The selected person **can** be both.)

Summary

Here is a summary of the key points of the addition rule:

1. To find $P(A \text{ or } B)$, first associate the word **or** with addition.
2. To find the value of $P(A \text{ or } B)$, add the number of ways A can occur and the number of ways B can occur, but be careful to add without double counting.

Complementary Events and the Addition Rule

We use \bar{A} to indicate that event A does not occur. Common sense dictates this principle: We are certain (with probability 1) that either an event A occurs *or* it does not occur, so it follows that $P(A \text{ or } \bar{A}) = 1$. Because events A and \bar{A} must be disjoint, we can use the addition rule to express this principle as follows:

$$P(A \text{ or } \bar{A}) = P(A) + P(\bar{A}) = 1$$

Rule of Complementary Events

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\bar{A})$$

Example: Sleepwalking (1 of 2)

Based on a journal article, the probability of randomly selecting someone who has sleepwalked is 0.292, so $P(\text{sleepwalked}) = 0.292$ (based on data from “Prevalence and Comorbidity of Nocturnal Wandering in the U.S. General Population,” by Ohayon et al., **Neurology**, Vol. 78, No. 20). If a person is randomly selected, find the probability of getting someone who has **not** sleepwalked.

Example: Sleepwalking (2 of 2)

Solution

Using the rule of complementary events, we get

$$\begin{aligned}P(\text{has **not** sleepwalked}) &= 1 - P(\text{sleepwalked}) \\ &= 1 - 0.292 = 0.708\end{aligned}$$

The probability of randomly selecting someone who has not sleepwalked is 0.708.

Multiplication Rule

Notation

$P(A \text{ and } B) = P(\text{event } A \text{ occurs in a first trial and event } B \text{ occurs in a second trial})$

$P(B | A)$ represents the probability of event B occurring after it is assumed that event A has already occurred.

Intuitive Multiplication Rule

To find the probability that event A occurs in one trial and event B occurs in another trial, multiply the probability of event A by the probability of event B , but **be sure that the probability of event B is found by assuming that event A has already occurred.**

Formal Multiplication Rule

$$P(A \text{ and } B) = P(A) \cdot P(B | A)$$

Independence and the Multiplication Rule

- Independent
 - Two events A and B are **independent** if the occurrence of one does not affect the **probability** of the occurrence of the other. (Several events are independent if the occurrence of any does not affect the probabilities of the occurrence of the others.) If A and B are not independent, they are said to be **dependent**.

Example: Screening Drugs and the Basic Multiplication Rule (1 of 6)

50 test results from the subjects who use drugs are shown below:

Positive Test Results : 45

Negative Test Results : 5

Total : 50

Example: Screening Drugs and the Basic Multiplication Rule (2 of 6)

- a. If 2 of these 50 subjects are randomly selected **with replacement**, find the probability the first selected person had a positive test result and the second selected person had a negative test result.
- b. Repeat part (a) by assuming that the two subjects are selected **without** replacement.

Example: Screening Drugs and the Basic Multiplication Rule (3 of 6)

Solution

a. With Replacement: First selection (with 45 positive results among 50 total results):

$$P(\text{positive test result}) = \frac{45}{50}$$

Second selection (with 5 negative test results among the same 50 total results):

$$P(\text{negative test result}) = \frac{5}{50}$$

Example: Screening Drugs and the Basic Multiplication Rule (4 of 6)

Solution

We now apply the multiplication rule as follows:

P (1st selection is positive and 2nd is negative)

$$= \frac{45}{50} \cdot \frac{5}{50} = 0.0900$$

Example: Screening Drugs and the Basic Multiplication Rule (5 of 6)

Solution

b. Without Replacement: Without replacement of the first subject, the calculations are the same as in part (a), except that the second probability must be adjusted to reflect the fact that the first selection was positive and is not available for the second selection. After the first positive result is selected, we have 49 test results remaining, and 5 of them are negative.

The second probability is therefore $\frac{5}{49}$.

Example: Screening Drugs and the Basic Multiplication Rule (6 of 6)

Solution

P (1st selection is positive and 2nd is negative)

$$= \frac{45}{50} \cdot \frac{5}{49} = 0.0918$$

Sampling

In the world of statistics, sampling methods are critically important, and the following relationships hold:

- Sampling **with replacement**: Selections are **independent** events.
- Sampling **without replacement**: Selections are **dependent** events.

Treating Dependent Events and Independent

5% Guideline for Cumbersome Calculations

When sampling without replacement and the sample size is no more than 5% of the size of the population, treat the selections as being **independent** (even though they are actually dependent).

Example: Drug Screening and the 5% Guideline for Cumbersome Calculations (1 of 3)

Assume that three adults are randomly selected **without replacement** from the 247,436,830 adults in the United States. Also assume that 10% of adults in the United States use drugs. Find the probability that the three selected adults all use drugs.

Example: Drug Screening and the 5% Guideline for Cumbersome Calculations (2 of 3)

Solution

Because the three adults are randomly selected without replacement, the three events are dependent, but here we can treat them as being independent by applying the 5% guideline for cumbersome calculations. The sample size of 3 is clearly no more than 5% of the population size of 247,436,830.

Example: Drug Screening and the 5% Guideline for Cumbersome Calculations (3 of 3)

Solution

We get $P(\text{all 3 adults use drugs})$

$$= P(\text{first uses drugs and second uses drugs and third uses drugs})$$

$$= P(\text{first uses drugs}) \cdot P(\text{second uses drugs}) \cdot P(\text{third uses drugs})$$

$$= (0.10)(0.10)(0.10)$$

$$= 0.00100$$

There is a 0.00100 probability that all three selected adults use drugs.

Redundancy: Important Application of the Multiplication Rule

The principle of **redundancy** is used to increase the reliability of many systems.

Our eyes have passive redundancy in the sense that if one of them fails, we continue to see. An important finding of modern biology is that genes in an organism can often work in place of each other. Engineers often design redundant components so that the whole system will not fail because of the failure of a single component.

Example: Airbus 310; Redundancy for Better Safety (1 of 5)

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will work. For example, the Airbus 310 twin-engine airliner has three independent hydraulic systems, so if any one system fails, full flight control is maintained with another functioning system.

Example: Airbus 310; Redundancy for Better Safety (2 of 5)

For this example, we will assume that for a typical flight, the probability of a hydraulic system failure is 0.002.

a. If the Airbus 310 were to have one hydraulic system, what is the probability that the aircraft's flight control would work for a flight?

b. Given that the Airbus 310 actually has three independent hydraulic systems, what is the probability that on a typical flight, control can be maintained with a working hydraulic system?

Example: Airbus 310; Redundancy for Better Safety (3 of 5)

Solution

a. The probability of a hydraulic system failure is 0.002, so the probability that it does **not** fail is 0.998. That is, the probability that flight control can be maintained is as follows:

$$\begin{aligned}P(1 \text{ hydraulic system } \mathbf{does\ not\ fail}) &= 1 - P(\text{failure}) \\ &= 1 - 0.002 \\ &= 0.998\end{aligned}$$

Example: Airbus 310; Redundancy for Better Safety (4 of 5)

Solution

b. With three independent hydraulic systems, flight control will be maintained if the three systems do not all fail. The probability of all three hydraulic systems failing is $0.002 \cdot 0.002 \cdot 0.002 = 0.000000008$.

It follows that the probability of maintaining flight control is as follows:

$P(\text{it does not happen that all three hydraulic systems fail}) = 1 - 0.000000008 = 0.999999992$

Example: Airbus 310; Redundancy for Better Safety (5 of 5)

Interpretation

With only one hydraulic system we have a 0.002 probability of failure, but with three independent hydraulic systems, there is only a 0.000000008 probability that flight control cannot be maintained because all three systems failed.

By using three hydraulic systems instead of only one, risk of failure is decreased not by a factor of $\frac{1}{3}$, but by a factor of $\frac{1}{250,000}$. By using three independent hydraulic systems, risk is dramatically decreased and safety is dramatically increased.

Summary of Addition Rule and Multiplication Rule

- **Addition Rule for $P(A \text{ or } B)$:** The word **or** suggests addition, and when adding $P(A)$ and $P(B)$, we must add in such a way that every outcome is counted only once.
- **Multiplication Rule for $P(A \text{ and } B)$:** The word **and** for two trials suggests multiplication, and when multiplying $P(A)$ and $P(B)$, we must be sure that the probability of event B takes into account the previous occurrence of event A .