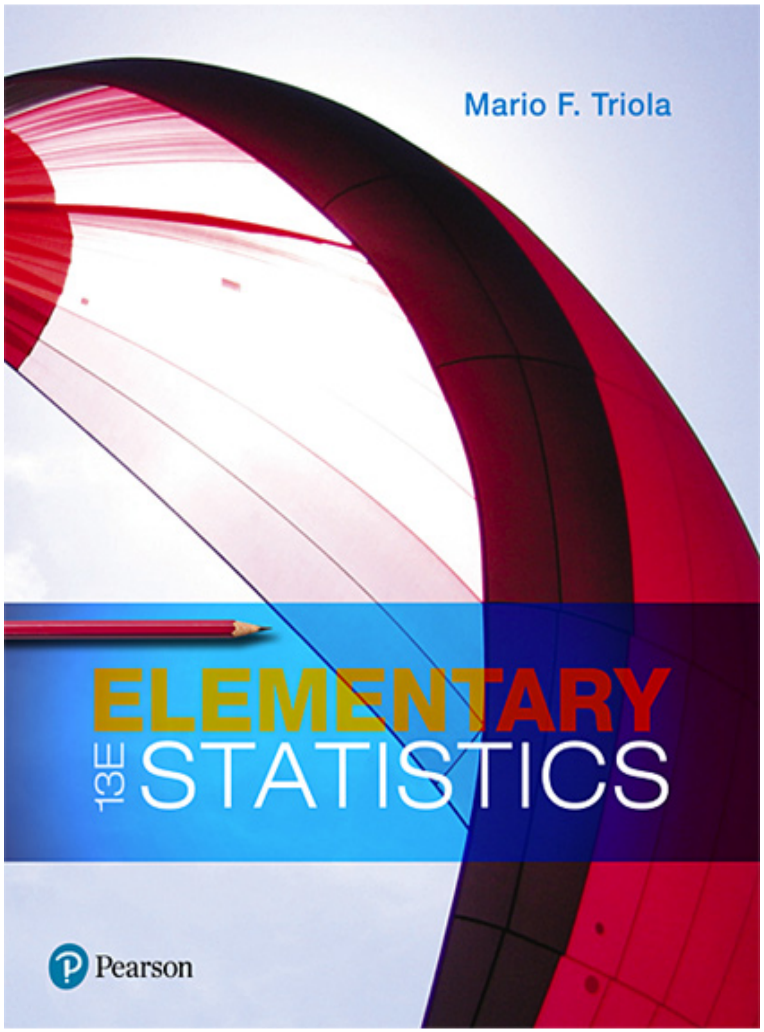
Elementary Statistics

Thirteenth Edition



Chapter 9

Inferences from Two Samples

Inferences from Two Samples

9-1 Two Proportions

9-2 Two Means: Independent Samples

9-3 Two Dependent Samples (Matched Pairs)

9-4 Two Variances or Standard Deviations

Key Concept

This section presents methods for testing hypotheses and constructing confidence intervals involving the mean of the differences of the values from two populations that are dependent in the sense that the data consist of matched pairs. The pairs must be matched according to some relationship, such as before/after measurements from the same subjects or IQ scores of husbands and wives.

Good Experimental Design

When designing an experiment or planning an observational study, using dependent samples with matched pairs is generally better than using two independent samples.

Inferences About Differences from Matched Pairs: Objectives

1. **Hypothesis Test:** Use the differences from two dependent samples (matched pairs) to test a claim about the mean of the population of all such differences.
2. **Confidence Interval:** Use the differences from two dependent samples (matched pairs) to construct a confidence interval estimate of the mean of the population of all such differences.

Inferences About Differences from Matched Pairs: Notation for Dependent Samples

- d = individual difference between the two values in a single matched pair
- μ_d = mean value of the differences d for the **population** of all matched pairs of data
- $\bar{d}$ = mean value of the differences d for the paired **sample** data
- s_d = standard deviation of the differences d for the paired **sample** data
- n = number of **pairs** of sample data

Inferences About Differences from Matched Pairs: Requirements

1. The sample data are dependent (matched pairs).
2. The matched pairs are a simple random sample.
3. Either or both of these conditions are satisfied: The number of pairs of sample data is large ($n > 30$) or the pairs of values have differences that are from a population having a distribution that is approximately normal. These methods are **robust** against departures for normality, so the normality requirement is loose.

Inferences About Differences from Matched Pairs: Test Statistic for Dependent Samples (with H_0: $\mu_d = 0$) (1 of 2)

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

***P*-Values:** *P*-values are automatically provided by technology or the *t* distribution in Table A-3 can be used.

Inferences About Differences from Matched Pairs: Test Statistic for Dependent Samples (with H_0: $\mu_d = 0$) (2 of 2)

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

Critical Values: Use Table A-3 (*t* distribution).
For degrees of freedom, use df = $n - 1$.

Inferences About Differences from Matched Pairs: Confidence Intervals for Dependent Samples

$$\bar{d} - E < \mu_d < \bar{d} + E$$

where $E = t_{\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}$ (Degrees of freedom: df = $n - 1$.)

Procedures for Inferences with Dependent Samples

1. Verify that the sample data consist of dependent samples (or matched pairs), and verify that the requirements in the preceding slides are satisfied.
2. Find the difference *d* for each pair of sample values.
3. Find the value of $\bar{d}$ and s_d.
4. For hypothesis tests and confidence intervals, use the same *t* test procedures used for a single population mean.

Equivalent Methods

Because the hypothesis test and confidence interval in this section use the same distribution and standard error, they are **equivalent** in the sense that they result in the same conclusions. Consequently, a null hypothesis that the mean difference equals 0 can be tested by determining whether the confidence interval includes 0.

Example: Are Best Actresses Generally Younger Than Best Actors? (1 of 9)

Data lists ages of actresses when they won Oscars in the category of Best Actress, along with the ages of actors when they won Oscars in the category of Best Actor. The ages are matched according to the year that the awards were presented. This is a small random selection of the available data so that we can better illustrate the procedures of this section. Use the sample data with a 0.05 significance level to test the claim that for the population of ages of Best Actresses and Best Actors, the differences have a mean less than 0.

Actress (years)	28	28	31	29	35
Actor (years)	62	37	36	38	29
Difference *d*	−34	−9	−5	−9	6

Example: Are Best Actresses Generally Younger Than Best Actors? (2 of 9)

Solution

Requirement Check (1) The samples are dependent because the values are matched by the year in which the awards were given. (2) The pairs of data are randomly selected. We will consider the data to be a simple random sample. (3) Because the number of pairs of data is $n = 5$, which is not large, we should check for normality of the differences and we should check for outliers. There are no outliers, and a normal quantile plot would show that the points approximate a straight-line pattern with no other pattern.

All requirements are satisfied.

Example: Are Best Actresses Generally Younger Than Best Actors? (3 of 9)

Solution

Step 1: The claim that the differences have a mean less than 0 can be expressed as $\mu_d < 0$ year.

Step 2: If the original claim is not true, we have $\mu_d \geq 0$ year.

Step 3: The null hypothesis must express equality and the alternative hypothesis cannot include equality, so we have

H_0: $\mu_d = 0$ year H_1: $\mu_d < 0$ year (original claim)

Example: Are Best Actresses Generally Younger Than Best Actors? (4 of 9)

Solution

Step 4: The significance level is $\alpha = 0.05$.

Step 5: We use the Student *t* distribution.

Step 6: Before finding the value of the test statistic, we must first find the values of $\bar{d}$ and s_d. We use the differences (−34, −9, −5, −9, 6) to find these sample statistics: $\bar{d} = -10.2$ years and $s_d = 14.7$ years.

Example: Are Best Actresses Generally Younger Than Best Actors? (5 of 9)

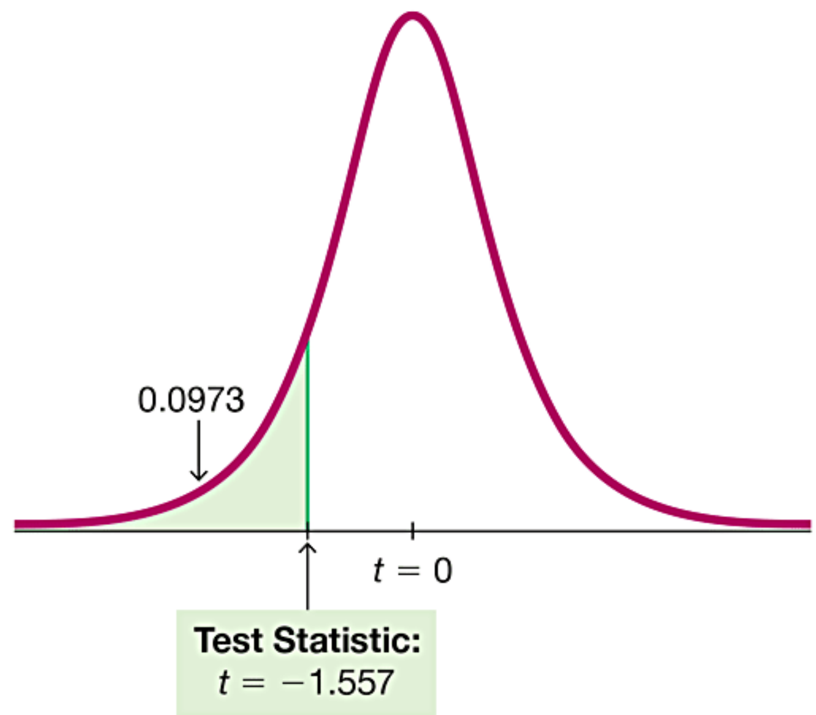
Solution

Step 6 (con't): Using these sample statistics and the assumption from the null hypothesis that $\mu_d = 0$ year, we can now find the value of the test statistic. (The value of $t = -1.557$ is obtained if unrounded values of $\bar{d}$ and s_d are used; technology will provide a test statistic of $t = -1.557$.)

Example: Are Best Actresses Generally Younger Than Best Actors? (6 of 9)

Solution

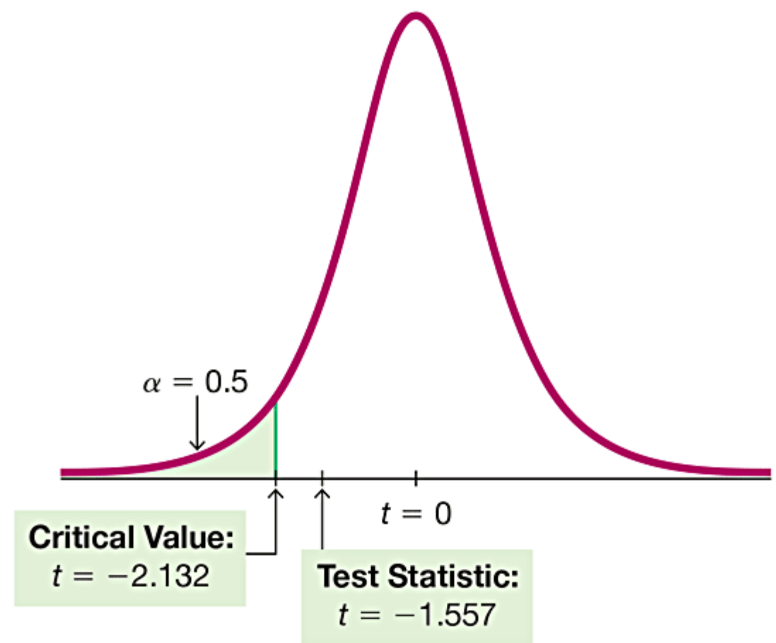
***P*-Value Method** Because we are using a *t* distribution, we refer to Table A-3 for the row with df = 4 and we see that the test statistic $t = -1.552$ corresponds to an "Area in One Tail" that is greater than 0.05, so *P*-value > 0.05. Technology would provide *P*-value = 0.0973.



Example: Are Best Actresses Generally Younger Than Best Actors? (7 of 9)

Solution

Critical Value Method Refer to Table A-3 to find the critical value of $t = -2.132$ as follows: Use the column for 0.05 (Area in One Tail), and use the row with degrees of freedom of $n - 1 = 4$. The critical value $t = -2.132$ is negative because this test is left-tailed where all values of t are negative.



Pearson

Example: Are Best Actresses Generally Younger Than Best Actors? (8 of 9)

Solution

Step 7: If we use the *P*-value method, we fail to reject H_0 because the *P*-value is greater than the significance level of 0.05. If we use the critical value method, we fail to reject H_0 because the test statistic does not fall in the critical region.

Example: Are Best Actresses Generally Younger Than Best Actors? (9 of 9)

Interpretation

We conclude that there is not sufficient evidence to support $\mu_d < 0$. There is not sufficient evidence to support the claim that for the population of ages of Best Actresses and Best Actors, the differences have a mean less than 0. There is not sufficient evidence to conclude that Best Actresses are generally younger than Best Actors.

Example: Confidence Interval for Estimating the Mean of the Age Differences (1 of 4)

Using the same sample data, construct a 90% confidence interval estimate of μ_d, which is the mean of the age differences. By using a confidence level of 90%, we get a result that could be used for the previous hypothesis test. (Because the hypothesis test is one-tailed with a significance level of $\alpha = 0.05$, the confidence level should be 90%.)

Example: Confidence Interval for Estimating the Mean of the Age Differences (2 of 4)

Solution

REQUIREMENT CHECK The solution for previous example includes verification that the requirements are satisfied.

The Statdisk display shows the 90% confidence interval. It is found using the values of $\bar{d} = -10.2$ years, $s_d = 14.7$ years, and $t_{\frac{\alpha}{2}} = 2.132$ (found from Table A-3 with $n - 1 = 4$ degrees of freedom and an area of 0.10 divided equally between the two tails).

Statdisk

Sample size, n: 5
Difference Mean, d: -10.2
Difference Standard Deviation, sd: 14.65264
Test Statistic, t: -1.5566
Critical t: -2.1318
P-Value: 0.0973

90% Confidence interval:
$-24.16971 < \mu d < 3.769712$

Example: Confidence Interval for Estimating the Mean of the Age Differences (3 of 4)

Solution

We first find the value of the margin of error *E*.

$$E = t_{\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}} = 2.132 \cdot \frac{14.7}{\sqrt{5}} = 14.015853$$

We now find the confidence interval.

$$\bar{d} - E < \mu_d < \bar{d} + E$$

$$-10.2 - 14.015853 < \mu_d < -10.2 + 14.015853$$

$$-24.2 \text{ years} < \mu_d < 3.8 \text{ years}$$

Example: Confidence Interval for Estimating the Mean of the Age Differences (4 of 4)

Interpretation

We have 90% confidence that the limits of −24.2 years and 3.8 years contain the true value of the mean of the age differences. In the long run, 90% of such samples will lead to confidence interval limits that actually do contain the true population mean of the differences. The confidence interval includes the value of 0 year, so it is possible that the mean of the differences is equal to 0 year, indicating that there is no significant difference between ages of Best Actresses and Best Actors.

Alternative Method Used When Population Is Not Normal and $n \leq 30$

For the following requirement: The number of pairs of sample data is large ($n > 30$) or the pairs of values have differences that are from a population having a distribution that is approximately normal, if that condition is violated, we can use the “Bootstrap Procedure for a Confidence Interval Estimate of a Parameter” included in Section 7-4.