

Exam 2 – Probability

Study Guide & Review



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Chapters 4 & 5

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General Exam Info

Exams are a way for you to show me what you have learned (and please show all your steps so I can see this!) and to give you a sense of accomplishment! They are meant to be challenging and not just homework problems with the numbers changed. I really want to prepare you for university level math classes—so some exams may be longer or more challenging than others. Remember that I do grade fairly and my goal is to push you to succeed and excel in this class.

- Attendance required for all exams—I do NOT drop the lowest exam score.
- **“Make-up Exams”** are given only in extreme cases and at instructor’s discretion; a student is allowed at most one make-up exam. (Documentation must be provided for the instructor to even consider a make up exam. This means you would need a doctor’s note, etc.) A “Make-Up Exam” means you will be allowed to replace the missing score with the percentage you earn on the final exam. Please contact your instructor as soon as possible should there be a problem.
- **Your student ID is required for all exams.**
- During the exams—you will be required to leave your backpack and all non-test items at the front of the room, including cell phones and smart watches. Only your pencil/eraser and calculator will be allowed during the exam, and there will be a calculator check. Should you need to leave during the exam please ask for permission first before leaving and leave your cell phone with me. Not doing these things could result in a 0 on your exam.
- Once the exam is graded and returned, any problem you would like me to revisit must be brought to my attention by the next class session.
- Always keep your exams!

Exam 1 Date & Time

- | | | |
|----------|-------------------|-------------------|
| • Exam 2 | Thursday, April 4 | 1:15 pm – 2:35 pm |
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ALSO, after the exam:

- As the test is only 80 minutes long, we will have a 10 minutes break, then continue with new material.
- I take attendance at the end of class on test days.

Exam 2 Specific Info

- Almost all questions have multiple parts
- You will need a calculator (only Ti83/84 allowed)—you won’t be able to use your phone.
- You need to know what all the various terms in **bold** mean, but you don’t need to memorize definitions. I’m not going to ask you to “Define ...”. Instead I ask you questions that use those terms.
- Material covered: Sections 4.1, 4.2, 4.3, 5.1, & 5.2
- Old material: You still need to know Chapter 2 and Chapter 3, especially Chapter 3 material.
- I may ask you multiple choice/true-false/circle the right answer/fill-in the blanks/short response types of questions (you may need to provide examples).
- $(M \rightarrow E)$ **“Math to English”** If you see this, it just means that I would like you to write your answer in a complete sentence and include all relevant **units**.

Chapter 4

§4.1

• Set Theory

- **set notation:** { things }.
- **sample space** S .
Ex: $S = \{HH, HT, TH, TT\}$,
Ex: $S = \{1, 2, 3, 4, 5, 6\}$, etc

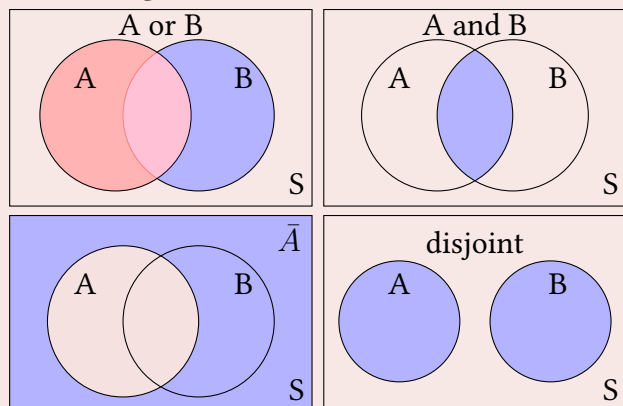
• events: A, B, C, \dots

- Ex: $E = \{2, 4, 6\}$ = “rolling an even #”

• simple events

- Ex: $A = \{5\}$ = “rolling a 5”

• Venn Diagrams



• Probability

- Classical/Theoretical Probability

$$P(A) = \frac{\text{\#outcomes in } A}{\text{\#outcomes in } S}$$

- Relative Frequency Probability

$$P(A) = \frac{\text{\#occurrences of } A}{\text{\#of trials } n}$$

- Subjective Probability

- Rounding rule for probability
- Probabilities can be written as decimals, fractions, and percentages. But decimals are preferred.

$$0 \leq P(A) \leq 1$$

- Impossible events: $P(A) = 0$

- Certain events: $P(A) = 1$

§4.2

• Independent vs Dependent events

• Probability Rules

- Sum Rule

General:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Disjoint: $P(A \text{ or } B) = P(A) + P(B)$

- Multiplication Rule

A, B indep. $\rightarrow P(A \text{ and } B) = P(A) * P(B)$

A, B dep. $\rightarrow P(A \text{ and } B) = P(A) * P(B|A)$

- Complement Rule $P(\bar{A}) = 1 - P(A)$

- Probability of Disjoining sets: $P(A \text{ and } B) = 0$ when A and B are disjoint

• Using Probability to determine significance:

- $P(x \text{ or more}) \leq 0.05 \rightarrow x$ is significantly high
- $P(x \text{ or less}) \leq 0.05 \rightarrow x$ is significantly low

§4.3

• “At least one” Trick

$$P(\text{at least one}) = 1 - P(\text{none})$$

• Conditional Probability

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\frac{\text{\#A and B}}{\text{\#S}}}{\frac{\text{\#A}}{\text{\#S}}}$$

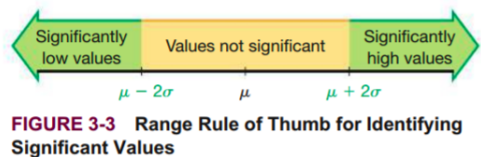
Chapter 4

For the following I may ask you multiple choice/true-false/circle the right answer types of questions but mostly you’ll see free-response questions similar to the worksheet problems:

- Be able to distinguish between classical/theoretical, relative frequency, and subjective probabilities.
- Be able to identify events as impossible or certain.
- Be able to use set notation to express: sample space, events, and simple events
- Be able to compute probabilities from the definition and using the Sum rule, Multiply rule, and Complement rule
- Understand when events are independent vs dependent and how this effects probabilities using conditional probabilities
- Study all of the examples from the worksheets carefully! Be prepared to information given to you in table.

§5.1

- **Random Variables X**
- Discrete vs Continuous Random Variables
- **Probability Distributions** for a RV X
 - as a list of all probabilities
 - as a table of values
 - as a histogram
 - **Requirements**
 1. $\sum P(x) = 1$
 2. $0 \leq P(x) \leq 1$
- Formulas for Probability Distributions
 - mean: $\mu = \sum [x \cdot P(x)]$
 - variance: $\sigma^2 = \sum (x - \mu)^2 \cdot P(x)$
 - variance: $\sigma^2 = \sum [x^2 \cdot P(x)] - \mu^2$
 - standard deviation: $\sigma = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2}$
- **Expectation** $E(X) = \sum [x \cdot P(x)]$
 - key point: the “long run behavior” or “many/large sample”
 - expectation is also called the mean. So the mean tells us the long run expectations
- **Range Rule of Thumb:**
 - **Significantly Low:** value $< \mu - 2\sigma$
 - **Significantly High:** value $> \mu + 2\sigma$
 - **Not Significant:** $\mu - 2\sigma < \text{value} < \mu + 2\sigma$



§5.2

- **Binomial Probability Distributions**
 - **Requirements**
 1. fixed number of trials
 2. trials are independent
 3. **only two** outcomes (“success” or “failure”)
 4. probability of success of each trial remains the same
 - **Notation**
 - * n total number of trials
 - * p be the probability of a single success
 - * q be the probability of a single failure. (NOTE: $q = 1 - p$)
 - * x be the number of successful trials of X . So notice that x can take values from 0 up to n , i.e. $x = 0, 1, 2, 3, \dots, n$.
- **Binomial Probability Formula**
 - Let $P(X = x)$ denote the **probability of exactly x successful trails out n in a binomial probability distribution**, then $P(X = x) = {}_n C_x p^x q^{n-x}$
 - $P(X = x) = \text{binompdf}(n, p, x)$
- Formulas for Binomial Distribution
 - $\mu = np$ $\sigma^2 = npq$ $\sigma = \sqrt{npq}$
 - Recall: the range rule of thumb
- Cumulative Probabilities
 - $01 \dots x \dots n$ $P(X \leq x) = \text{binomcdf}(n, p, x)$
 - $01 \dots x \dots n$ $P(X < x) = \text{binomcdf}(n, p, x - 1)$
 - $01 \dots x \dots n$ $P(X \geq x) = 1 - \text{binomcdf}(n, p, x)$
 - $01 \dots x \dots n$ $P(X \leq x) = 1 - \text{binomcdf}(n, p, x)$

For the following I may ask you multiple choice/true-false/circle the right answer types of questions but mostly you’ll see free-response questions similar to the worksheet problems:

- Be able to distinguish between discrete and continuous random variables.
- Be able to determine whether or not information given represents a probability distribution.
- Be able to build a probability distribution for a given random variable. (Ex: a) X = number of Heads that come up out of 4 coin flips, b) X = number of girls that a family has if they plan to have 4 children)
- Be able to recognize if a probability distribution is binomial so that you can use binompdf or binomcdf
- Be able to compute probabilities of binomial distributions for a single value of x and multiple values of x using the rules of probability from Ch 4 (including “at least one”, “or”, “and” etc).
- Be able to create a table for a binomial probability distribution using your calculator
- Be able to compute expectations and know their significance
- Be able to compute the mean, standard deviation, and variance given a probability distribution
- Be able to recognize if a value is significantly low or high using the range rule of thumb

Calculator Skills you must know

- **STATS** Edit lists, clear lists
- **1-Var Stats** use to compute mean, standard deviation, 5 number summary
- **2nd+VARS** Distributions → binompdf(n,p,x) and binomcdf(n,p,x)
- **2nd+ENTER** Trick!

Exam 2 Practice Problems

- Chapter 4 Quiz Review: p.178: #1, 2, 4 – 10 all;
- Chapter 4 Review Excersizes: p.178-179: #1 – 10 all; 12, 13;
- Chapter 5 Quiz Review: p.220: #1 – 10 all;
- Chapter 5 Review Excersizes: p.220-221: #1 – 5 all; 7, 8, 9;

Formula Sheet for Exam 2

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- $P(A \text{ and } B) = P(A) * P(B)$
- $P(A \text{ and } B) = P(A) * P(B|A)$
- $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$
- $\mu = \sum [x \cdot P(x)]$
- $\sigma^2 = \sum (x - \mu)^2 \cdot P(x)$
- $\sigma = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2}$
- $E(X) = \sum [x \cdot P(x)]$