Chapter 12: Analysis of Variance

Section 12.1: One-Way ANOVA — Stat 50

CONTINGENCY TABLES

Def ***One-way analysis of variance (ANOVA)*** is a method of testing the equality of three or more population means by analyzing sample variances.

One-way ANOVA is used with data categorized with one factor, or treatment, so there is one characteristic used to separate the sample data into the different categories.

What are the hypotheses going to look like?

H_0: All of the $\mu's$ are equal H_1: Not all of the $\mu's$ are equal

Then find sample means for each population $(\bar{x}_1, \bar{x}_2, \ldots . \bar{x}_k)$

- If all of the $\bar{x}_k's$ are CLOSE together, then ______________________________ H_0.
- If all of the $\bar{x}_k's$ are FAR APART, then ________________________________ H_0.

Steps for Hypothesis Test for ANOVA

Check Requirements:

- Each population must have a normal distribution.
- Each population must have the same variance, σ^2.
- Samples are independent, simple random samples

Step 2: Level of Significance

Step 1: Hypotheses

$$H_0: \ \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$$

$$H_1: \ \textit{At least one of the means is different from the others}$$

ALWAYS ____________-TAILED TEST!

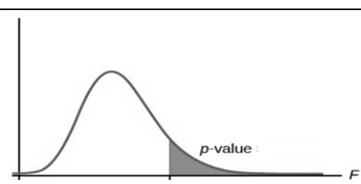
What to find:

Source	df	Sum of Squares	Mean Square
Factor	$k-1$	$n\sum_{j=1}^{k}(\bar{x}_j - \bar{x})^2$	$MS(factor) = \dfrac{SS(factor)}{k-1}$
Error	$k(n-1)$	$\sum_{j=1}^{k}\sum_{i=1}^{n}(x_{ij} - \bar{x}_j)^2$	$MS(error) = \dfrac{SS(error)}{k(n-1)}$

Step 3: Test Statistic $F = \dfrac{MS(factor)}{MS(error)} = \dfrac{\text{variance between samples}}{\text{variance within samples}}$

Step 4: Use P-Value Method

$Fcdf(F_0, UB, k-1, k(n-1))$



$$\begin{cases} \textit{Reject } H_0 \ \sim \ \text{if } \textit{P-value} \le \alpha \\ \textit{Fail to Reject } H_0 \ \sim \ \text{if } \textit{P-value} > \alpha \end{cases}$$

Step 5: Write a CONCLUSION either rejecting or failing to reject H_0

Ex: Three groups of randomly selected students were given a different dosage of statistioprazole, a new drug that supposedly increases a student's ability to perform statistics. Each group was given a statistics pretest, then two groups were given a certain level of statistioprazole for two weeks while another group took a placebo. Then all groups took a post test (out of 10). The data is assumed to be from populations that are normally distributed with equal variances. Test the claim that the mean test scores are the same for each treatment with $\alpha = 0.05$.

Null and Alternative Hypothesis

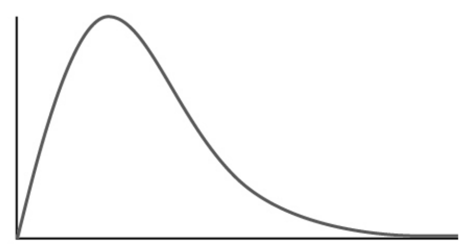
Placebo Group	10 mg Group	20 mg Group
6	9	8
8	6	10
5	6	9
3	8	9
$n_1 =$ $\bar{x}_1 =$ $s^2{}_1 =$	$n_2 =$ $\bar{x}_2 =$ $s^2{}_2 =$	$n_3 =$ $\bar{x}_3 =$ $s^2{}_3 =$

Variance *between* samples	Variance *within* samples
$ns_{\bar{x}}^2 =$	$s_p^2 =$

Test Statistic

$$F = \frac{ns_{\bar{x}}^2}{s_p^2}$$

P-value



Decision about Null Hypothesis

Conclusion

Graphing Calculator (TI-83 or 84)

Instructions:
(a) Input each sample into a separate list
(b) STAT $\Rightarrow$ TESTS $\Rightarrow$ H: ANOVA(
(c) Enter Lists separated by commas $\Rightarrow$ ENTER

Ex: A consumer agency randomly selected auto drivers who had similar driving records, cars, and insurance policies. The provided table gives the monthly insurance premiums (in dollars) by these drivers insured with one of four insurance companies.

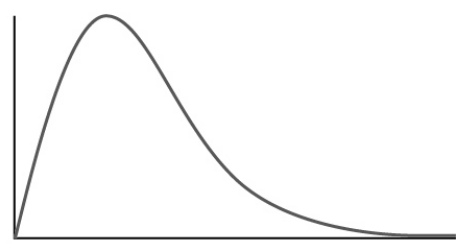
State Farm	75	83	102	90	84	77	91	78
Geico	81	92	78	72	94	85	93	77
Farmers	70	82	75	67	91	74	72	83
AAA	78	86	84	68	88	77	65	70

At the 0.05 significance level, test the claim that the mean monthly insurance premiums are equal, assuming that the populations are normally distributed with equal variances.

Null and Alternative Hypothesis

Test Statistic

P-value



Decision about Null Hypothesis

Conclusion