Chapter 8: Hypothesis Testing

Section 8.1: Basics of Hypothesis Testing — Stat 50

GOAL: Make a decision about p *or* μ based on $\hat{p}$ *or* $\bar{x}$ using ________________.

Chapter 7: Find a sample then **estimate** whether the population fits within a certain interval.	**Chapter 8**: Given a past claim of the parameter, we will ______________ whether or not it has changed.

STRUCTURE OF A HYPOTHESIS TEST

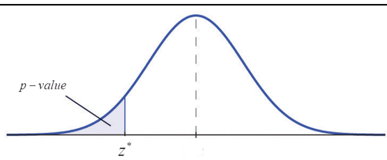
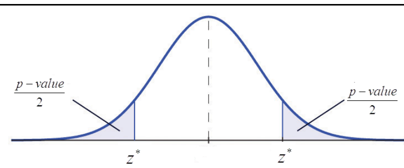
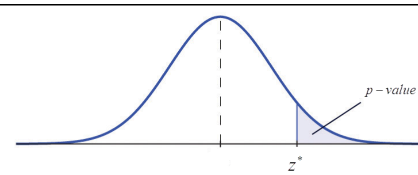
1) Make an assumption about reality	•
2) Look at a sample evidence	•
3) Determine whether it contradicts our assumption.	•

We won't be 100% certain, we will just be able to tell if sample data ____________________ a statement or not.

HYPOTHESES STATEMENTS

NULL HYPOTHESIS ( )	ALTERNATIVE HYPOTHESIS ( )
A statement of __________________________, no effect, no difference and is assumed true until evidence indicates otherwise.	A statement that we are trying to find evidence to ___________ instead of ___________.

THREE TYPES OF HYPOTHESIS TESTS

LEFT-TAILED	TWO-TAILED	RIGHT-TAILED
$H_{0:}\ parameter = \#$ $H_1: parameter$ _________	$H_{0:}\ parameter = \#$ $H_1: parameter$ _________	$H_{0:}\ parameter = \#$ $H_1: parameter$ _________
		

Ex: **1)** What's the parameter? **2)** What do "they say"? **3)** What do we think? **4)** What type of test?

The packaging on a light bulb says it should last 500 hours. Consumer Reports wants to know if the mean lifetime is actually less than that.	The standard deviation of the rate of return for some mutual funds is 0.08%. A manager believes the standard deviation might be higher than that.	According to a Gallup poll in 2008, 80% of Americans felt satisfied with the way things are going in their lives. A researcher wonders if the percentage is different now.
1) 2) and 3) 4)	1) 2) and 3) 4)	1) 2) and 3) 4)

TWO possibly correct conclusions:

1) We decide there **is** evidence to support H_1	2) We decide there **is NOT** enough evidence to support H_1

Ex: Historically, Jimbo's pizza had a mean delivery time of 48 minutes. After getting a new pizza oven, he takes a sample of 50 orders and finds that the mean delivery time is now 45 minutes, which makes Jimbo think that the mean delivery time has been reduced.

State Jimbo's hypotheses in statistical notation:	State the conclusion if the **null is rejected:**	State the conclusion if the **null is *not* rejected:**

Four possible outcomes (2 errors)

Example: In a court case
H_0: the defendant is innocent
H_1: the defendant is guilty

		Truth about the Population (**Reality**)	
		H_0 is true	H_0 is false
Decision Based On Sample **(Our Conclusion)**	Fail to Reject H_0	Conclude __________ when __________	Conclude __________ when __________
	Reject H_0	Conclude __________ when __________	Conclude __________ when __________

***NOTE:** The defendant is NEVER declared INNOCENT!!

Type I and Type II Errors

Type I error: The mistake of rejecting the null hypothesis when it is actually true.
The symbol α (alpha) is used to represent the probability of such an error.

Type II error: The mistake of failing to reject the null hypothesis when it is actually false.
The symbol β (beta) is used to represent the probability of such an error

Ex: On average, it used to take 30 minutes to find parking, but we think we have sufficient evidence to say that the time has decreased. But, in fact, the true parking time is still 30 minutes. What kind of error did we make?

Note: The majority is a number or percentage equaling more than ________________ of a total. To test a claim about a majority, your null hypothesis will be $p =$ ___________.

<u>Ex</u>: A Gallup survey reports that 57% of 504 randomly selected gun owners support stricter gun laws. Test the claim that a majority of gun owners favor stricter gun laws. Write out the hypotheses for this example. What would a Type II error be in this scenario?

<u>Ex</u>: Your company markets a computerized device to test a patient's mean resting heart rate. Based on the sample results, the device determines whether there is significant evidence that the patient's mean resting heart rate is greater than 100 beats per minute. If so, your company recommends that the person seeks medical attention.

a. State appropriate null and alternative hypotheses in this setting.

H_0:

H_1:

b. Which error is worse for your company?

	H_0 is true	H_0 is false
Fail to Reject H_0		**Type II Error**
Reject H_0	**Type I Error**	

Type II Error → Seek Med Attention? YES NO
Did They Need It? YES NO

Type I Error → Seek Med Attention? YES NO
Did They Need It? YES NO

We will NOT know 100% if our conclusion of our Hypothesis Test is ___________, but we can assign _______________________________ to making Type I and Type II Errors when we complete a hypothesis test.

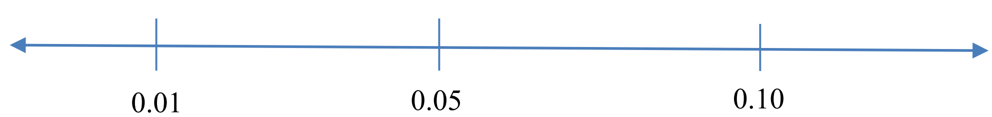
Level of Significance	The probability of making a Type I Error. In other words, we take a sample that makes H_0 look WRONG when it's actually TRUE.

Note: As you __________________ the probability of one type of error, then the probability of the other type _________________.

Choosing a Significance Level

Typically the significance level, α is given to be greater than _______________ and less than _______________.

When a Type I error is…



Hypothesis Testing: Claim about a Proportion

Requirements

1. The sample observations are a simple random sample.
2. The conditions for a binomial distribution are satisfied.
3. If $n\,p \geq 5$ and $n\,q \geq 5$, then the normal distribution can be used to approximate the distribution of sample proportions with mean $\mu = n\,p$ and standard deviation $\sigma = \sqrt{n\,p\,q}$.

<table>
<tr><th colspan="2">Steps for a Hypothesis Test When Applied to Testing p</th></tr>
<tr><td>Pre-Step: Check Requirements
● It is a valid ________________________ sample
● The requirements are met to use the needed distribution.</td><td></td></tr>
<tr><td>Step 1: Hypotheses
H_0: $p = p_0$
H_1: ____________ or ____________ or ___________</td><td>Step 2: Level of Significance</td></tr>
<tr><td colspan="2">Step 3: Test Statistic (Find a z-score, t-value or X^2 value)</td></tr>
<tr><td colspan="2">Step 4: Find a critical value or P-value to check using either the Critical Value or P-value method.</td></tr>
<tr><td colspan="2">Step 5: Make a decision and draw a conclusion</td></tr>
</table>

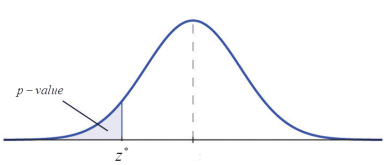
Null and Alternative Hypothesis

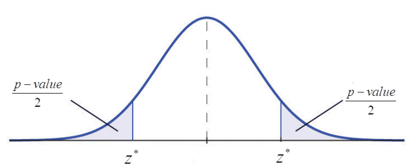
Left-Tailed	Two-Tailed	Right-Tailed
$H_0: \; p = p_0$ $H_1: \; p < p_0$	$H_0: \; p = p_0$ $H_1: \; p \neq p_0$	$H_0: \; p = p_0$ $H_1: \; p > p_0$

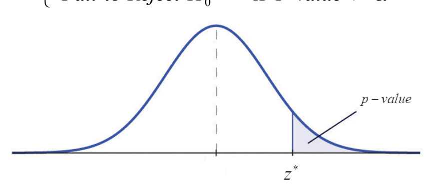
P-Value Method

Decision

$$\begin{cases} \text{Reject } H_0 \sim \text{ if } P\text{-value} \leq \alpha \\ \text{Fail to Reject } H_0 \sim \text{ if } P\text{-value} > \alpha \end{cases}$$



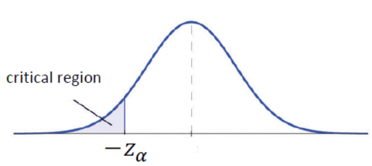


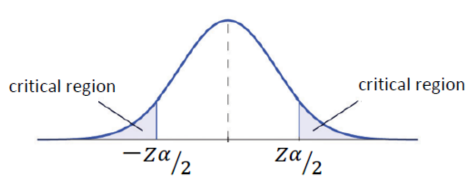


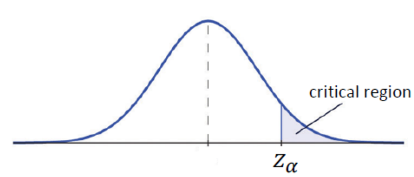
Critical Value Method

Decision

$$\begin{cases} \text{Reject } H_0 \sim \text{ if } z^* \text{ lies in the critical region} \\ \text{Fail to Reject } H_0 \sim \text{ if } z^* \text{ doesn't lie in the critical region} \end{cases}$$





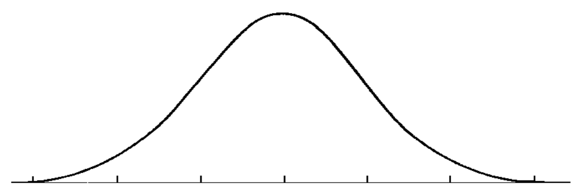


Ex: According to the Census Bureau, 8.8% of the U.S. population had no health insurance coverage in 2017. Suppose that in a recent random sample of 1200 Americans, 130 had no health insurance. Use a 0.02 significance level to test the claim that the current percentage of Americans who have no health insurance coverage is greater than 8.8%. Use the ____________________ method.

Null and Alternative Hypothesis

Test Statistic

P-value/Critical Region



Decision about Null Hypothesis

Conclusion

Identify the Type I error

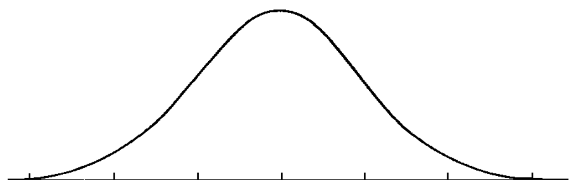
Identify the Type II error

Ex: According to NPR, in 2016 32.1% of adults aged 18-34 lived at home with their parents. A sociologist recently randomly surveyed 500 people aged 18-34 and found that 143 of them did. At $\alpha = 0.05$, do we think the proportion has changed? Use the ____________________ method.

Null and Alternative Hypothesis

Test Statistic

P-value/Critical Region



Decision about Null Hypothesis

Conclusion

Graphing Calculator (TI-83 or 84)

Instructions: (a) STAT $\Rightarrow$ TESTS $\Rightarrow$ 1-PropZTest

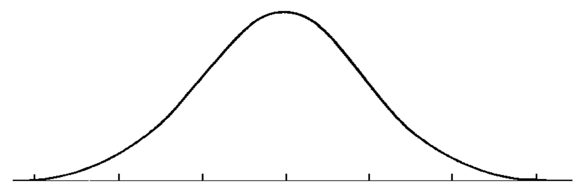
(b) Enter
$$\begin{cases} p_0 = \text{population proportion stated in } H_0 \\ x = \text{number of successes} \\ n = \text{number of trials} \\ prop \sim \text{alternative hypothesis} \end{cases}$$

Ex: In a 2016 Gallup poll, 34% of people said that it was morally acceptable to clone animals. In 2017, a survey found that 192 out of 600 randomly selected people believed that it was morally acceptable to clone animals. Use a 0.10 significance level to test the claim that less than 34% of all adults say that it is morally acceptable to clone animals. Use the ____________________ method.

Null and Alternative Hypothesis

Test Statistic

P-value/Critical Region



Decision about Null Hypothesis

Conclusion

Big Question: Is this ____________________________ ________________________________?

Def ***Statistically Significant*** When observed results are unlikely under the assumption that the null hypothesis is true and we reject the null hypothesis.

Hypothesis Testing: Claim About a Mean (σ Not Known)

Requirements

1. The sample is a simple random sample.
2. The value of the population standard deviation σ is not known.
3. Either or both of the given conditions are satisfied: $\begin{cases} \text{The population is normally distributed} \\ \text{or} \\ n > 30 \end{cases}$

Ex: Recall the logic behind a hypothesis test: Say $H_0: \mu = 10$ and $H_1: \mu < 10$

If we take a sample and find the point estimate…

- $\bar{x} = 10$ then we ____________________ H_0
- $\bar{x} > 10$ then we ____________________ H_0
- $\bar{x} < 10$ by "a little", then we ____________________ H_0
- $\bar{x} < 10$ by "a lot", then we ____________________ H_0

Steps for *a* Hypothesis Test When Applied to Testing μ	
Pre-Step: *Check Requirements* ● It is a valid ______________________ sample ● The requirements are met to use the needed distribution.	
Step 1: *Hypotheses* $H_0: \mu = \mu_0$ H_1: ___________ or ___________ or ___________	***Step 2:*** *Level of Significance*
Step 3: *Test Statistic* (Find a z-score, t-value or X^2 value)	
Step 4: *Find a Critical Value or P-Value to check using either the Critical Value or P-value method.*	
Step 5: *Make a decision and draw a conclusion.*	

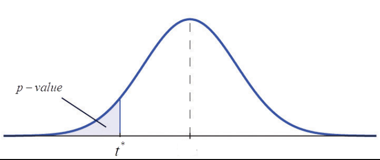
Null and Alternative Hypothesis

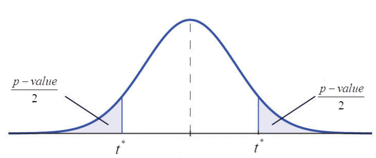
Left-Tailed	Two-Tailed	Right-Tailed
$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$

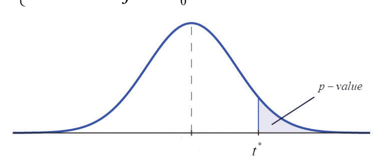
P-Value Method

Decision

$\begin{cases} \textit{Reject } H_0 \sim \text{ if } \textit{P-value} \le \alpha \\ \textit{Fail to Reject } H_0 \sim \text{ if } \textit{P-value} > \alpha \end{cases}$



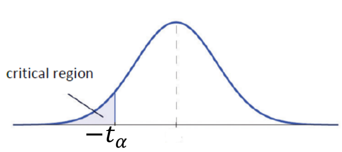


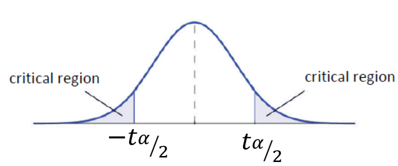


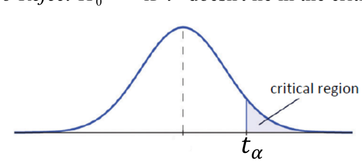
Critical Value Method

Decision

$\begin{cases} \textit{Reject } H_0 \sim \text{ if } t^* \text{ lies in the critical region} \\ \textit{Fail to Reject } H_0 \sim \text{ if } t^* \text{ doesn't lie in the critical region} \end{cases}$





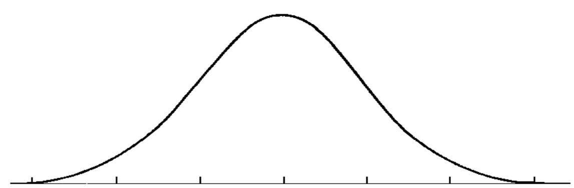


<u>Ex</u>: Kaiser Foundation hospital claims that the mean waiting time for patients to be seen in the emergency room is 20 minutes. A random sample of 40 patients produced a mean waiting time of 18.5 minutes and a standard deviation of 4.0 minutes. Use a 0.10 significance level to test the claim that the mean waiting time is equal to 20 minutes. Use the ____________________ method.

Null and Alternative Hypothesis

Test Statistic

P-value/Critical Region



Decision about Null Hypothesis

Conclusion

Identify the Type I error

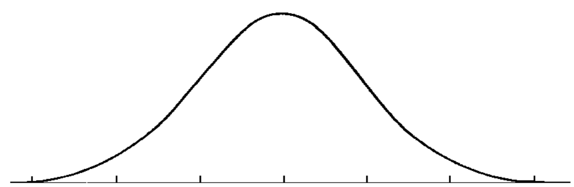
Identify the Type II error

Ex: According to the Bureau of Labor Statistics, the mean amount of money spent by a household on alcohol in the US is \$565 per year. A church group wants to check this claim and took a random sample of 45 households and found that mean amount spent on alcohol per year was \$520 with a standard deviation of \$167. Test the church group's claim that the mean amount of money spent on alcohol per year is less than \$565. Use the ____________________ method.

Null and Alternative Hypothesis

Test Statistic

P-value/Critical Region



Decision about Null Hypothesis

Conclusion

GRAPHING CALCULATOR (TI-83 OR 84)

Instructions: (a) STAT $\Rightarrow$ TESTS $\Rightarrow$ T-Test

(b) Enter
$$\begin{cases} \mu_0 = \text{population mean stated in } H_0 \\ s = \text{sample standard deviation} \\ \overline{x} = \text{sample mean} \\ n = \text{sample size} \\ \mu \sim \text{alternative hypothesis} \end{cases}$$

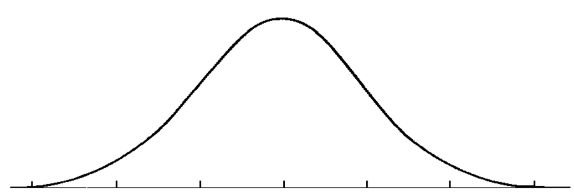
Ex: The Instagram handle @getfollowers claims that they can increase the number of followers someone has on Instagram. In March 2015 the mean number of followers for a US teen was 150, so a random sample of 12 US teens with 150 followers was taken. The following is the number of followers, which is normally distributed, these US teens had after they paid @getfollowers for their help. Using a 0.01 level of significance, determine whether @getfollowers is effective at increasing the number of Instagram followers. Use the _____________________ method.

160	200	152	150	145	151	162	158	156	149	154	170

Null and Alternative Hypothesis

Test Statistic

P-value/Critical Region



Decision about Null Hypothesis

Conclusion