Stat 50 - Elementary Statistics — Winter 2020

Exam 3 – Normal and Sampling Dist., CLT, Confidence Intervals

Study Guide & Review

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Chapters 6 & 7 & 8.1

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General Exam Info

Exams are a way for you to show me what you have learned (and please show all your steps so I can see this!) and to give you a sense of accomplishment! They are meant to be challenging and not just homework problems with the numbers changed. I really want to prepare you for university level math classes–so some exams may be longer or more challenging than others. Remember that I do grade fairly and my goal is to push you to succeed and excel in this class. I often give hints in class as to exam problems (another great reason to come to class!), and I will post study guides along with the best way to review for each exam.

- Four exams are given during the semester–check our schedule for the exact dates.
- **Attendance required for all exams** and there are **"No Make-up Exams"** for any reason. However, I may replace the lowest exam score, regardless of the reason, with your final exam score provided the final exam score is higher than your lowest exam score and all assignments are turned-in on time.
- Your valid **PCC student ID** or a valid **government ID** is REQUIRED for all exams.
- During the exams–you will be required to leave your backpack and all non test items at the front of the room, including cell phones and smart watches. Only your pencil/eraser and calculator will be allowed during the exam, and there will be a calculator check. Should you need to leave during the exam please ask for permission first before leaving and leave your cell phone with me. Not doing these things could result in a 0 on your exam.
- Once the exam is returned, any problem you would like me to revisit must be brought to my attention by the next class session.
- **Always keep your exams!**

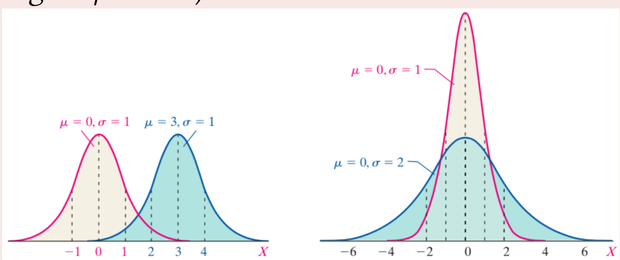
Exam 3 Specific Info

- The exam is scheduled for the end of the class period, the last 80 minutes.
- Almost all questions have multiple parts
- This test will be closed book, no notes.
- You will need a calculator (only Ti83/84 allowed)–you won't be able to use your phone.
- You need to know what all the various terms in **bold** mean, but you don't need to memorize definitions. I'm not going to ask you to "Define ... ". Instead I ask you questions that use those terms.
- **Material covered: Sections 6.1, 6.2, 6.3, 6.4, 7.1, 7.2, & 8.1**
- Old material: You still need to know Chapters 1-5, especially Chapter 4 and 5 material. Basically: don't forget to study from Exam 1 and Exam 2 (and also the study guides!).
- $(M \to E)$ **"Math to English"** If you see this, it just means that I would like you to write your answer in a complete sentence and include all relevant **units**.
- Types of Questions to expect:
  - True-False
  - Circle the right answer
  - Fill-in the blank
  - Multiple-choice
  - Short response

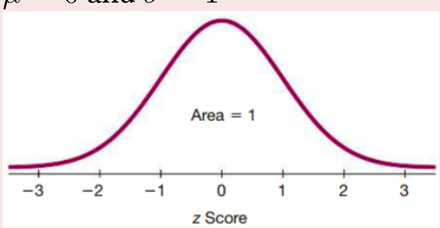
Chapter 6

§6.1

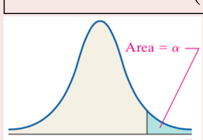
- X a continuous, RV.
- **density curve** associated to X
- **Important:** area under density curve = probability
- **Normal Distribution**
  - bell-shaped, symmetric, center at μ, standard deviation σ
  - there are lots of normal distributions (depending on μ and σ)



- **Standard Normal Distribution**
  - $\mu = 0$ and $\sigma = 1$
  -



- **Finding probabilities given z-scores:**
  - $P(a < X < b) =$ normalcdf(a, b, μ, σ)
  - Can replace "$<$" or "$>$" with "$\leq$" or "$\geq$" for normal distributions (also, any continuous distribution)
  - **Symmetry Trick:**
    $P(X < \mu) = 0.5$ and $P(X > \mu) = 0.5$
  - If $a < \mu$, compute using symmetry trick: $P(X < a) = 0.5 - P(a < X < \mu)$
  - If $a < \mu$, compute using symmetry trick: $P(X > a) = P(a < X < \mu) + 0.5$
  - There's many more cases, but draw a picture!
- **Finding x-value given area/probability:**
  - $x =$ invNorm$(\alpha, \mu, \sigma, \text{RIGHT})$
  -



  - **Critical Value z_α**
    $z_\alpha =$ invNorm$(\alpha, \mu, \sigma, \text{RIGHT})$
- If use older TI calculators: you must use left tail:
  $z_\alpha =$ invNorm$(1 - \alpha, \mu, \sigma)$

§6.2

- Examples of Normal Distributions
- Important section, uses all the theory from §6.1
- Will need the formulas for z-scores from earlier:
  $z = \dfrac{x - \mu}{\sigma}$ and $x = \mu + z \cdot \sigma$

§6.3

- **Sampling Distribution**
  - **Sampling distribution of proportion**: 1) approximately normal, 2) sample proportion **targets** the value of the true population proportion
  - **Sampling distribution of mean**: 1) approximately normal, 2) sample mean **targets** the value of the true population mean
- **Estimators**
  - **Unbiased:** sample stat targets pop parameter
    Examples: $\hat{p}, \bar{x}, s^2$
  - **Biased:** sample stat DOESN'T target pop parameter
    Examples: Med, Range, s

§6.4

- **Central Limit Theorem:**
  - **Requirements:** X original distribution is normal or sample size $n > 30$.
  - CLT says: sampling distribution of sample means (samplING dist.) can be approximated by normal distribution with $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$
- **WHEN TO USE CLT vs Normal Dist?**
  - when computing probability of **one random sample**: use §6.1, 6.2 ie use original μ and σ.
  - when computing probability of **many random samples**: use CLT and samplING dist. USE $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. Must check: if $n > 30$ or original X is normal.

Chapter 6

For the following I may ask you multiple choice/true-false/circle the right answer types of questions but mostly you'll see free-response questions similar to the worksheet problems:

- Study all of the examples from the worksheets carefully!
- Know what a density curve is and the properties of probability distributions
- Know the important fact that area=probability under a density curve
- Know the important properties of normal distributions, including the empirical rule (from previous chapters)
- Be able to compute z-scores
- Be able to compute probabilities given z-scores for many examples including a picture of the correct shaded area (there will lots of problems on this–see worksheet problems and hw problems). You will use your calculator with normalcdf. Especially using the symmetry properties discussed in class.
- Be able to compute a z-score given an area/probability. You will use your calculator with invNorm.
- Know what a sampling distribution is and how it is different than a regular distribution.
- Know what biased and unbiased estimators are.
- Be able to state the requirements, the givens, and the conclusion of the CLT (as in the worksheet).
- Be able to recognize when to use the CLT vs regular normal distribution when solving probability problems.

Chapter 7

§7.1

- **point estimate**
- **Confidence Interval for Population Proportion**:
  - **CI**: $(\hat{p} - E, \hat{p} + E)$
  - point estimate for population proportion is $\hat{p}$
  - $\hat{p} = \frac{x}{n}$ = $\frac{\#\text{success}}{\#\text{ sample size}}$
  - **CL**: $CL = 1 - \alpha$, $\alpha = 1 - CL$, $\alpha/2$
  - **Critical Value**:
    $z_{\alpha/2} = \text{invNorm}(\alpha/2, 0, 1, \text{RIGHT})$
    uses standard normal distribution
    draw a picture!!
  - **Error**: $E = z_{\alpha/2}\sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$, where $\hat{q} = 1 - \hat{p}$.
- **Requirements for CI for pop. prop.:**
  1. SRS: simple random sample
  2. Conditions for binomial distributions: 4 conditions to check!
  3. $nq \geq 5$, i.e. "at least 5 successes and 5 failures"
- **MEANING**:
  A CI of 95% tells us that *the process* of finding the CIs should in the long run result in CIs that contain the true pop. prop. 95% of the time, that is 95% of the CIs will contain the true pop. prop.
- **Calculator**
  - STAT → TESTS → **1-PropZInt**
- **Determining sample size**:
  - $\hat{p}$ **known** → use $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2}$
  - $\hat{p}$ **unknown** → use $n = \frac{[z_{\alpha/2}]^2 0.25}{E^2}$
  - round up!

§7.2

- **point estimate**
- **Confidence Interval for Population Mean**:
  - **CI**: $(\bar{x} - E, \bar{x} + E)$
  - point estimate for population mean is $\bar{x}$
  - $\bar{x} = \frac{\sum x}{n}$, or using calculator **1-Var-Stats**
  - **CL**: $CL = 1 - \alpha$, α, $\alpha/2$
  - **Critical Value**: $t_{\alpha/2} = \text{invT}(1 - \alpha/2, \text{df})$
    *uses **student t-distribution***
    - bell-shaped, centered at $\mu = 0$, fatter tails
    - depends on **degrees of freedom**, df
      $df = n - 1$
    - As n increases, t-distribution approaches standard normal distribution
    - For $n > 30$, can assume t-distribution is standard normal distribution

    draw a picture!!
  - **Error**: $E = t_{\alpha/2}\frac{s}{\sqrt{n}}$.
- **Requirements for CI for pop. mean:**
  1. SRS: simple random sample
  2. normality
  3. sample size $n > 30$
- **MEANING**:
  A CI of 95% tells us that *the process* of finding the CIs should in the long run result in CIs that contain the true pop. mean. 95% of the time, that is 95% of the CIs will contain the true pop. mean.
- **Calculator**
  - STAT → TESTS → **1-TInterval**
  - Either summary statistics given, or enter data into a list
- **Determining sample size**:
  - σ **known** → use $n = \left[\frac{z_{\alpha/2} \cdot \sigma}{E}\right]^2$
  - round up!

Chapter 7

For the following I may ask you multiple choice/true-false/circle the right answer types of questions but mostly you'll see free-response questions similar to the worksheet problems:

- Study all of the examples from the worksheets carefully!
- Be able to solve problems involving confidence intervals in **5 steps** (as in worksheets):
  1. Find **point estimate** (either $\hat{p}$ or $\bar{x}$)
  2. Find **critical value** (either $z_{\alpha/2}$ or $t_{\alpha/2}$)

3. Find the **error** (either $E = z_{\alpha/2}\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}}$ or $E = t_{\alpha/2}\frac{s}{\sqrt{n}}$)
4. Find the **confidence interval** (either $(\hat{p} - E, \hat{p} + E)$ or $(\bar{x} - E, \bar{x} + E)$)
5. $^\star(M \to E)^\star$ **Interpretation**

- Pay close attention to step 5 and practice writing your answers in a complete sentence.
- Pay close attention to the units given in the problem.
- Be able to determine the sample size needed for a confidence interval either for population proportion ($\hat{p}$ known or unknown) or mean (σ known).

Chapter 8

§8.1

- **Hypothesis Testing**:
  - **Null Hypothesis** H_0
    a null hypothesis will almost always be an "="
  - **Alternative Hypothesis** H_1 (or H_A)
    alt hypothesis will be: $<$, $>$, or $\neq$
- **Type of Test**:
  - **Left-Tailed**
  - **Two-Tailed**
  - **Right-Tailed**

- **Type of Errors**:
  - **Type I Error**
  - **Type II Error**
  -

		Reality: H_o Is True	Reality: *H_0 is False* H_1 Is True
Conclusion	Do Not Reject H_o	Correct Conclusion	Type II Error
Conclusion	Reject H_o	Type I Error	Correct Conclusion

- **Levels of Significance**:
  - $\alpha = P(\text{Type I Error}) = P(\text{Reject } H_0 | H_0 \text{is True})$
  - $\beta = P(\text{Type II Error}) = P(\text{Fail to Reject } H_0 | H_0 \text{is False})$
  - α and β are **inversely related**: if $\alpha \downarrow$ then $\beta \uparrow$, and if $\alpha \uparrow$ then $\beta \downarrow$

Chapter 8.1

For the following I may ask you multiple choice/true-false/circle the right answer types of questions but mostly you'll see free-response questions similar to the worksheet problems:

- Study all of the examples from the worksheets carefully in Section 8.1!
- Be able to identify and state the null and alternative hypotheses in problems using correct math notation.
- Be able to write correct complete sentences based on conclusions of hypothesis tests.
- Be able to identify type I or II errors in word problems and explain them in complete sentences.

Calculator Skills you must know

- STATS Edit lists, clear lists
- 1-Var Stats use to compute mean, standard deviation, 5 number summary
- 2nd+VARS Distributions
  - normalcdf(a,b,μ, σ)
  - invNorm(α, μ, σ,LEFT/CENTER/RIGHT) older Ti: can only use left tails. Use invNorm($1-\alpha, \mu, \sigma$)
- 2nd+ENTER Trick!
- Confidence Intervals:
  - for pop. proportion use: STAT $\rightarrow$ TESTS $\rightarrow$ **1-PropZInt**
  - for pop. mean use: STAT $\rightarrow$ TESTS $\rightarrow$ **1-TInterval**
    Either summary statistics given, or enter data into a list

Practice Test

Note: solutions to all review exercises are available at the back of the book!

- **Chapter 6**
  - Chapter Quick Quiz, p.291-292: # 1-9 all;
  - Review Exercises, p.292-294: # 1-4 all, 5a,b, 6, 7a,b;
  - Cumulative Review Exercises, p. : #
- **Chapter 7**
  - Chapter Quick Quiz, p.350: # 2,-6 all, 9;
  - Review Exercises, p.351-352: # 1-4 all, 5a,e, 7;
  - Cumulative Review Exercises, p. : #
- **Chapter 8**
  - Chapter Quick Quiz, p.407-408: #3;
  - Review Exercises, p.408-409: #1$a, b, e, 5, 8a$;
  - Cumulative Review Exercises, p. : #

Formula Sheet for Exam 3

- $\text{normalcdf}(a, b, \mu, \sigma)$
- $\text{invNorm}(\alpha, \mu, \sigma, \text{TAIL})$
- $z = \dfrac{x-\mu}{\sigma}$ and $x = \mu + z \cdot \sigma$
- $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$

- $CL = 1-\alpha$, $\alpha = 1 - CL$, $\alpha/2$
- $E = \mathbf{z}_{\alpha/2}\sqrt{\dfrac{\hat{p}\cdot\hat{q}}{n}}$
- $n = \dfrac{[z_{\alpha/2}]^2\hat{p}\hat{q}}{E^2}$
- $n = \dfrac{[z_{\alpha/2}]^2 0.25}{E^2}$
- $\text{invT}(1-\alpha/2, \text{df})$
- $E = \mathbf{t}_{\alpha/2}\dfrac{s}{\sqrt{n}}$
- $n = \left[\dfrac{z_{\alpha/2}\cdot\sigma}{E}\right]^2$